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MEMBERSHIP

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

AN ASYMMETRIC PROBABILITY FUNCTION

By J. J. Slade,[1] Jr., Esq.

Synopsis

In engineering problems that require statistical analysis, such as studies of rainfall and run-off, the data are frequently too meager to allow the use of the elaborate methods developed by Karl Pearson, or the Danish statisticians; and yet, the asymmetries in the frequency distributions associated with these problems are usually sufficiently marked to place the investigation in a field definitely outside that of the simple Gaussian theory. Unfortunately, where the basic probability function is other than the "normal curve", the determination of the constants involved, as well as the graduation of the data, generally becomes a difficult task. Furthermore, unless the data are very complete, the final results will seldom have meaning because of the magnitude of the probable errors of the parameters involved.

The purpose of this paper is to introduce a function that differs as little as it seems possible to let it differ from the "normal" in its general characteristics, while allowing it an unlimited degree of "skewness". At the same time, it is an easy curve to apply, as such things go, and one for the application of which existing tables may be used. The paper is subdivided into the following parts:

Section I contains a critical discussion of the various methods in use at present in the analysis of frequency distributions. The purpose of this section is, first of all, to contrast and to compare the function introduced with those already in use so that both its merits and defects will be exposed at the beginning. It will thus be seen to contain a justification for the introduction of a new function in a field that literally teems with such studies. It is hoped that this paper will also help the engineer to a better understanding of a subject the approach to which is barricaded by so many mathematical hurdles.

Note.—Discussion on this paper will be closed in January, 1935, *Proceedings*.

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for its use, although this is in no way comparable to the justification derived from experimental verification of the results obtained from its use. Under certain conditions it gives reliable information; this is a matter subject to controlled experiment. Its use is indicated whenever the following assumptions can be made regarding the variation of the entity considered[3]:

(a) The most probable (or most frequent) value of a large number of observations is their arithmetic mean.

(b) Small variations are more likely to occur than large ones.

These or equivalent assumptions relate to the variate; in addition, of course, the foundations of probability theory are assumed.

As long as the variation comes fairly well within this set the normal curve represents the data surprisingly well, and more significant than mere "fitting" of the given data is the fact that the information obtained by its use (as in extrapolating, for instance) is surprisingly accurate. The single parameter of "spread" seems adequate to take care of the possible configurations of such a set.

In engineering problems, however, these assumptions can seldom be made outside the case of direct and precise measurements. Usually the mean, the mode, and the median do not coincide.

Since the relation between negative and positive variations is not known when the symmetry that leads to the Gaussian distribution is lacking, a general frequency function cannot be developed entirely from *a priori* considerations. This leads to investigation of all the mathematical expressions that share with the probability function its characteristics properties, among which, *a fortiori*, must be the curve that is sought; and, since there is no method of generating all such curves, the investigator must finally be content to try as many of them as he can discover and to let experience be the final arbiter of the success of the search.

The extension of the theory to include skew probability functions has been the subject of much ingenious mathematical research.

2.—Pearson's Curves.—One of the first extensions of the theory, and perhaps still the most successful, is due to Karl Pearson,[4] who embodied the asymmetric assumptions regarding the frequency of the variations into the very general differential equation:

$$\frac{dy}{dx} = \frac{y(x + a)}{F(x)} \qquad \text{.......................(2)}$$

which merely states that the curve has a maximum at the point, $x = -a$, and that it approaches the axis of x asymptotically as the frequency vanishes. The normal curve is obtained from the family generated by Equation (2) simply by letting $F(x)$ = a constant. It is the simplest one of all; but this mode of generation is not altogether satisfactory because, whereas

[3] For a complete proof that the normal law follows from the principle of the arithmetic mean and the theorem on compound probabilities, see "Advanced Calculus", by E. B. Wilson, Ginn & Co., 1912, p. 386.

[4] "Frequency Curves and Correlation", by W. P. Elderton, Lond., 1927.

Equation (2) may contain all frequency curves, it is quite likely that it also contains many curves not even remotely related to frequency functions. Equation (2) does not provide a means of distinguishing between them; for, certainly, the problem of frequency distribution is not merely one of fitting, as well as possible, a curve to a set of data; there must be some kind of theory connected with it, as strong, if possible, as that which supports the normal curve. This is a desideratum not likely to be attained, but one which must be aimed at, nevertheless, in searching for general probability functions.

Another difficulty with Pearson's set-up is a practical one. The form of the function, $F(x)$, that appears in Equation (2) must be limited severely in order to integrate the equation at all. Pearson assumed that $F(x)$ can be developed in a Maclaurin series and that it is permissible to drop all the terms of the series after the square. This is merely a roundabout and unscientific manner of assuming that $F(x)$ is a quadratic.

However, the family of curves generated by Equation (2) under the assumption that,

$$F(x) = b_0 + b_1 x + b_2 x^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

has proved to be very useful. One of the principal objections to the family generated by Equation (2) is the variety of types into which it is divided. The criteria for this or that type are artificial and cumbrous. The objection is not altogether æsthetic; in order definitely to place a distribution in this or that category it is necessary to compute the fourth moment; and when the data are meager (as happens frequently in problems investigated by the engineer), a fourth moment is as meaningless as it is laborious to compute. Any accidental irregularity in the distribution is exaggerated dangerously in the higher moments. Even the third moment is quite unreliable when the observations are few. Nothing beyond the standard deviation is reliable for less than 150 items.

Of course, this family contains curves that may be specified completely by three moments (two when $\mu_1 = 0$; that is, when the origin is at the mean). One such is the curve classified as Type III; but this curve degenerates into an L-shaped figure for certain ranges of the parameters, which suffices to throw it out of the class of curves that represent possible homograde frequency distributions.

3.—*The General Theory.*—A general theory of frequency functions has been developed, following the researches originated by Laplace, which is extremely elegant mathematically, but quite impracticable in fact. (The best treatise, in English, on the theory of frequency curves as developed by the Danish school of statisticians, is that presented by Arne Fisher[5].) The theory is based on a fundamental property of frequency functions, namely, that if one has a set of observations, $o_1, o_2, \ldots, o_n$, and a set of numbers, $x_1, x_2, \ldots, x_m$, which represent the distinct values of the

[5] "The Mathematical Theory of Probability", by Arne Fisher, Macmillan, 1930.

observations, then, if $\phi(x)$ is the frequency function for the o's and $f(x)$ is any single-valued function at all, one may obviously write:

$$\sum_{r=1}^{r=n} f(o_r) = \sum_{r=1}^{r=m} f(x_r)\ \phi(x_r) \qquad (4)$$

Now, $\phi(x)$ is a function not only of the different values of the variate, but also of certain statistical parameters (the moments, for example, would be one such set); and, to a great extent, it is a matter of convenience what functions of the observations these parameters should be. For instance, they may be defined in this manner: Call the parameters, $\lambda_1, \lambda_2, \ldots, \lambda_k$, and take two convenient functions, $F(z, \lambda_1 \ldots, \lambda_k)$ and $\sum_{r=1}^{r=n} K(z, o_r)$. Now, if these two functions are equated and required to be equal for all values of z, a relation may be established in this manner between the parameters, λ_r, and the observations, o_r, because, by expanding both sides of the equation,

$$F(z, \lambda_1 \ldots ; \lambda_k) = \sum_{r=1}^{r=n} K(z, o_r) \qquad (5)$$

into series of powers of z and equating like powers, a relation is established between the parameters, λ_r, and the observations, o_r, that must hold for Equation (5) to be satisfied identically. By making use of Equation (4), Equation (5) may be written in this manner:

$$F(z, \lambda_1, \ldots, \lambda_k) = \sum_{r=1}^{r=m} K(z, x_r)\ \phi(x_r) \qquad (6)$$

or, passing to the limit in some reasonable manner:

$$F(z, \lambda_1, \ldots, \lambda_k) = \int_{-\infty}^{\infty} K(z, x)\ \phi(x)\ dx \qquad (7)$$

This is an integral equation in which, from Equation (5), both $F(z, \lambda)$ and $K(z, x)$ are known, and from which $\phi(x)$ may be determined by the rules established in the theory of such equations. The success with which efforts to solve Equation (7) will be rewarded, will depend on the ingenuity used in defining Equation (5).

Thiele obtained a solution of Equation (7) in terms of his famous semin-variants.[6] In Thiele's solution, $\phi(x)$ is given as a definite integral which, unfortunately, cannot be integrated; so that the net result is an elegant mathematical process which ultimately gets nowhere. Even if one could make direct use of Thiele's solution of Equation (7) there are certain

[6] "The Mathematical Theory of Probability", by Arne Fisher, 1930, p. 191.

circumstances that render it too general for ordinary purposes. In a manner somewhat analogous to the case of Pearson's differential equation, one may say that although Equation (7) certainly includes all frequency functions, it includes no theory of actual frequency functions. A fairly arbitrary function, $\phi(x)$, may satisfy Equation (7) without being remotely near an actual frequency function.

4.—The Gram-Charlier Series.—A much more practical development of the frequency function is the series named after Gram and Charlier. If one writes:

$$\phi_0(x) = \frac{1}{\sqrt{2\pi}} e^{-0.5x^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8a)$$

and,

$$\phi_k(x) = \frac{d^k \phi_0(x)}{dx^k} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8b)$$

then the frequency function may be represented thus,

$$\phi(x) = c_0\,\phi_0(x) + c_1\,\phi(x) + c_2\,\phi_2(x) + \ldots\ldots\ldots\ldots (9)$$

In this series the values of c_n may be obtained by making use of the orthogonal properties of the ϕ's, or they may be obtained by the usual method of least squares.[7] W. F. Osgood has written an elementary discussion of series of orthogonal functions and their connection with the principle of least squares.[8] Here, again, the criticism is that Equation (9) is too general; it represents a statement in mathematical analysis rather than one in the theory of frequency functions. In fact, if one is satisfied with some condition slightly less than an absolute mathematical approach to a limit, Equation (9) may be used to represent, very closely, quite fantastic functions—even discontinuous ones—provided they and their slopes vanish at infinity. Clearly, this includes some functions not at all connected with frequency functions.

Furthermore, if $\phi(x)$ deviates appreciably from $\phi_0(x)$ (that is, if it has a marked skew), it may take thousands of terms of Equation (9) to get even a fair representation—and this is a process which is practically impossible. The function, $\phi(x)$, need not deviate greatly from $\phi_0(x)$ before waves and negative frequencies appear by the use of a few terms of Equation (9), and they certainly do not belong in an actual frequency curve. The reason for these spurious phenomena will be obvious to the reader acquainted with Fourier analysis.

A generating function, $\phi_0(x)$, may be defined by a relation other than Equation (8) such that, to begin with, $\phi(x)$ and $\phi_0(x)$ will be fairly similar. Then, $\phi(x)$ may be closely represented by a few terms of Equation (9). This is so because each term represents a correction to the sum of the terms that precede it. Studies have been made of various forms of the generic func-

[7] "The Mathematical Theory of Probability", by Arne Fisher, 1930, pp. 199 *et seq.*
[8] "Advanced Calculus", by William Fogg Osgood, Macmillan, 1925.

tion, $\phi_0(x)$[9], but the objection to the use of Equation (9) must always remain: That this series, regardless of the form of the generic, $\phi_0(x)$, may be made to represent almost anything. It offers no theory of frequency functions.

5.—*Transformations of the Variate.*—For greatly skewed distributions a logarithmic transformation of the variate has sometimes proved useful.[10] This transformation really furnishes a new generic function, defined, according to Fisher, by

$$\phi_0(x) = \frac{1}{n\sqrt{2\pi}} e^{-\left(\log \frac{x-m}{n}\right)^2} \dots\dots\dots\dots\dots\dots (10)$$

to be used in Equation (9), and, therefore, it needs no further discussion than that of Article 4. The transformation has been suggested by the fact that when the skew of the frequency is great, that of the logarithms of the variate is more nearly normal. Special mention is made of this transformation because of its relation to the partly bounded function introduced in this paper. As objections to it other than those listed in Article 4 the following may be stated:

(*a*) The use of Equation (10) implies the solution of a cubic, distinct for each problem.

(*b*) After the transformation is accomplished one still has to determine the constants of the series by the method of least squares or by some equivalent process.

6.—*Empirical Curves.*—So-called empirical curves are not essentially different from any of the foregoing because, as a matter of fact, all the curves are empirical; but they suffer from the fact that they are usually devised for particular problems and the range of their applicability is generally quite limited. Furthermore, they are farther from an approximation of any theory of frequency functions than any of the foregoing. It is the integral, or duration, curve (for many purposes more important than the frequency function) which is often represented by means of purely arbitrary functions. Not much can be said in their favor except, perhaps, that they may be easy to use. The principal objection to their use lies in the fact that it is not possible to estimate the reliability of the parameters entering in them. The estimate of this reliability is an important phase of a statistical investigation often neglected in engineering applications.

7.—*Graphical Methods.*—The use of "probability" paper and other graphical devices is an undesirable practice. These methods convey to the eye a simplicity which does not really exist. The process of using these methods is not essentially different from that of fitting a curve to the data, except for the fact that one is more likely to fit the wrong curve graphically than analytically. A paper marked off in non-rectangular co-ordinates does to the data exactly what a flat map of a curved earth does to geography. One gets an exaggerated idea of the entire problem. A straight line that

[9] See, for instance, "Generalizations of the Normal Curve of Error", by Luis R. Salvosa, Dissertation, Univ. of Michigan, Edwards Bros., Inc., Ann Arbor, Mich.

[10] "The Mathematical Theory of Probability", by Arne Fisher, 1930, pp. 236 *et seq.*

seems to fit the data closely may not be a good fit at all when one remembers that small deviations in certain regions of the paper may represent quite large errors. Fitting a straight line to data that plot approximately as a straight line is like using the method of least squares without weighting for functional distortion.

One notable exception is presented by the closely allied semi-graphical methods of Dr. J. C. Kapteyn[11] and R. D. Goodrich, M. Am. Soc. C. E.[12] Their procedures are fundamentally sound. The weakness in their methods is due to the impossibility of computing the probable errors of the constants in their functions. There is no graphical process that gives an indication of these errors.

II—The Partly-Bounded Function

1.—Desiderata.—From the discussion of Section I it is seen that only one function furnishes something of a theory of variations, and that is the normal curve, Equation (1). The succeeding developments, although no doubt including the general theory of frequency functions within their scope, are far too general in character; in spite of all that may be said against them, the Pearson functions still remain the most useful. The introduction of a new function will be justifiable only in so far as it is able to call forth a minimum of objections that must surely be raised against it. The various requirements for it will now be listed and discussed. These requirements embody the writer's theory of the most general homograde probability function:

(*a*) It must be a simple extension of the normal curve; that is, the normal curve must be one of its possible forms, and for no range of the parameters that specify it must it lose its characteristic shape. It must be smooth, without waves, and without negative frequencies. It is true that waves will be exhibited legitimately by multi-modal distributions, but these must be considered as the superposition of several uni-modal distributions.

(*b*) It should be capable of assuming indefinite skewness. Since the factors that cause skewness in the frequency of a set of variations are not known, deviations from the normal may be assumed in some way to be connected with the fact that, usually, the variate is definitely bounded, the first deviation from the normal occurring from the fact that the variate has a minimum value, but no definite maximum. In a representative curve one may require that its skewness be a simple function of its end point. The function with one end point is here termed "partly bounded," that with two end points, "totally bounded".

(*c*) The curve must be specifiable completely by moments no higher than the third. Should it have more parameters than may be so specified, then it must be possible to find some extra-statistical means of determining them. Since the probable errors in the parameters increase greatly with the order of the moments necessary to specify them, it becomes necessary to search for some physical substitutes for these moments. Two such substitutes, it

11 "Skew Frequency Curves in Biology and Statistics", by J. C. Kapteyn and M. J. van Uven, Hoitsema Bros., Groningen, 1916.

12 *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 1 *et seq.*

will be shown, are the upper and lower bounds of the variation, when these bounds can be determined *a priori.*

(*d*) The curve must be simple to apply. Considering all the uncertainty that lies at the base of the theory of frequency functions, the information derived from their use is certainly not worth the expenditure of great labor in their application.

2.—The Partly Bounded Function.—Consider, first, the function:

$$y = a\, e^{-c^2 [\log d(x+b)]^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

which is defined from $x = -b$ to $x = \infty$. This function which is a first extension of the normal, admits of complete mathematical analysis. The study of it will be a guide in determining the parameters of the totally bounded function of Section IV.

Notice, in the first place, that Equation (11) is similar to the logarithmically transformed function, Equation (10), differing from it only in the power, c^2, to which it is raised. The difference is fundamental, however. It is just this added parameter, c, that makes Equation (11) more than a "normal" for the logarithms of the variate and which brings about a remarkable simplification of the formulas. It is this c, indeed, that makes Equation (11) a true generalization of Equation (1), as will be shown subsequently.

The discussion that follows immediately is a mathematical investigation of the properties of this function. By the usual method of moments[4] the parameters, a, b, c, and d, are determined in terms of the moments, μ_1, μ_2, μ_3, of the data and the total frequency, N. The investigation, which is given here for the first time, is developed with sufficient detail to enable the reader not well versed in mathematical manipulations to follow each step of the reasoning. It must be kept in mind that the analysis justifies the use of this curve only in so far as it shows that the function Equation (11) meets all the aforementioned requirements.

The reader who is willing to take for granted the accuracy of the following analysis may omit the remainder of this section and proceed to Section III, where the formulas obtained in Section II are tabulated and their application is discussed.

3.—The Parameters.—Let $S_\lambda =$ the power sums of the variations; that is, let,

$$S_\lambda = \Sigma\, x^\lambda, \; (\lambda = 0, 1, 2, 3) \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

so that, for instance, $S_0 = \Sigma\, x^0 = N$, the total frequency. Now, require these power sums to be equal to the corresponding moments of the function Equation (11). Thus,

$$S_\lambda = a \int_{-b}^{\infty} x^\lambda e^{-c^2 [\log d(x+b)]^2} dx \quad \ldots\ldots\ldots (13)$$

To integrate, let $z = \log d\,(x + b)$; then, $x = d^{-1} e^{z} - b$; $dx = d^{-1} e^{z}\, dz$;

and,

$$x^{\lambda} = d^{-\lambda}\,[e^{\lambda z} - \lambda\, b\, d\, e^{(\lambda-1)z} + \ldots (b\,d)^{\lambda}] \quad \ldots\ldots\ldots\ldots(14)$$

Substituting in Equation (13):

$$S_{\lambda} = \frac{a}{d^{\lambda+1}} \int_{-\infty}^{\infty} [e^{\lambda z} - \lambda\, b\, d\, e^{(\lambda-1)z} + \ldots]\, e^{z - c^2 z^2}\, dz \quad \ldots(15)$$

Equation (15) is a sum of integrals of the type, $\int_{-\infty}^{\infty} e^{kz - c^2 z^2}\, dz$.

Completing the square of the exponent in the integrand:

$$\int_{-\infty}^{\infty} e^{kz - c^2 z^2}\, dz = \int_{-\infty}^{\infty} e^{-c^2 (z^2 - \frac{k}{c^2} z + \frac{k^2}{4c^4} - \frac{k^2}{4c^4})}\, dz$$

$$= e^{\left(\frac{k}{2c}\right)^2} \int_{-\infty}^{\infty} e^{-c^2 \left(z - \frac{k}{2c^2}\right)^2}\, dz = \frac{\sqrt{\pi}}{c}\, e^{\left(\frac{k}{2c}\right)^2} \quad \ldots\ldots(16)$$

by a well-known result of the integral calculus.[13] Using this result in Equation (15):

$$S_{\lambda} = \frac{a}{d^{\lambda+1}} \frac{\sqrt{\pi}}{c} \left[e^{\left(\frac{\lambda+1}{2c}\right)^2} - \lambda\, b\, d\, e^{\left(\frac{\lambda}{2c}\right)^2} + \ldots (b\,d)^{\lambda}\, e^{\frac{1}{4c^2}} \right] \quad \ldots\ldots\ldots\ldots(17)$$

This gives four equations (one each for $\lambda = 0, 1, 2,$ and 3) with which to determine the constants, a, b, c, and d, in terms of the power sums, S_0, S_1, S_2, and S_3.

To solve Equation (17) let,

$$l = e^{\frac{1}{4c^2}};\quad M_{\lambda} = S_{\lambda} \frac{c\, d^{\lambda+1}}{a\sqrt{\pi}} \quad \ldots\ldots\ldots\ldots(18)$$

then Equation (17) may be written:

$$M_0 = l \quad \ldots\ldots\ldots\ldots(19a)$$

$$M_1 = l^4 - b\, d\, l \quad \ldots\ldots\ldots\ldots(19b)$$

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13 "Advanced Calculus", by F. S. Woods, p. 153, Ginn & Co., 1926.

$$M_2 = l^9 - 2\,b\,d\,l^4 + (b\,d)^2\,l \qquad \text{(19c)}$$

and,

$$M_3 = l^{16} - 3\,b\,d\,l^9 + 3\,(b\,d)^2\,l^4 - (b\,d)^3\,l \qquad \text{(19d)}$$

Solving for the powers of l:

$$l = M_0 \qquad \text{(20a)}$$

$$l^4 = M_1 + b\,d\,M_0 \qquad \text{(20b)}$$

$$l^9 = M_2 + 2\,b\,d\,M_1 + (b\,d)^2\,M_0 \qquad \text{(20c)}$$

and,

$$l^{16} = M_3 + 3\,b\,d\,M_2 + 3\,(b\,d)^2\,M_1 + (b\,d)^3\,M_0 \qquad \text{(20d)}$$

Now, $\dfrac{M_\lambda}{M_0} = d^\lambda\,\mu_\lambda$, ($\lambda = 1, 2, 3$), in which, according to the usual convention, μ_λ is the λth moment. Dividing Equations (20*b*), (20*c*), and (20*d*) by Equation (20*a*):

$$l^3 = d(\mu_1 + b) \qquad \text{(21a)}$$

$$l^8 = d^2(\mu_2 + 2b\mu_1 + b^2) \qquad \text{(21b)}$$

and,

$$l^{15} = d^3(\mu_3 + 3b\mu_2 + 3b^2\mu_1 + b^3) \qquad \text{(21c)}$$

a set from which a has been eliminated. The usual simplification may now be introduced, which in no way affects the generality of the results, obtained by choosing the origin at the mean so that $\mu_1 = 0$. Now, let $k = \dfrac{l^2}{d}$, so that Equations (21) become:

$$kl = b \qquad \text{(22a)}$$

$$k^2\,l^4 = \mu_2 + b^2 \qquad \text{(22b)}$$

and,

$$k^3\,l^9 = \mu_3 + 2\,b\,\mu_2 + b^3 \qquad \text{(22c)}$$

Eliminating b and the combination, kl, from Equations (22):

$$\mu_3 + \frac{\mu_2^{1.5}}{\sqrt{l^2 - 1}} = \frac{\mu_2^{1.5}}{(l^2 - 1)^{1.5}}\,(l^6 - 1)$$

that is,

$$\frac{\mu_3}{\mu_2^{1.5}} = \frac{l^6 - 1 - 3\,(l^2 - 1)}{(l^2 - 1)^{1.5}} = \sqrt{l^2 - 1}\,(l^2 + 2) \qquad \text{(23)}$$

an equation in which all the parameters of the curve except c have disappeared. Let $\beta = \dfrac{\mu_3}{\mu_2^{1.5}}$ and $t = \sqrt{l^2 - 1}$. Then, Equation (23) becomes:

$$t^3 + 3\,t - \beta = 0 \qquad \text{(24)}$$

a cubic with one real root equal to:

$$t = A^{\frac{1}{3}} + B^{\frac{1}{3}} \quad \text{.........................(25)}$$

in which, $A = \frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + 1}$; and $B = \frac{\beta}{2} - \sqrt{\frac{\beta^2}{4} + 1}$.

In terms of t all the parameters of the curve have simple expressions. For $l^2 = t^2 + 1$ and from Equation (18), $\frac{1}{2\,c^2} = \log\,(t^2 + 1)$; or,

$$c = [2 \log\,(t^2 + 1)]^{-0.5} \quad \text{.........................(26}$$

and,

$$b = k\,l = \frac{\sqrt{\mu_2}}{\sqrt{l^2 - 1}} = \frac{\sqrt{\mu_2}}{t} \quad \text{................(27)}$$

By definition, $k = \frac{l^2}{d}$. Therefore, from Equation (27):

$$d = \frac{t}{\sqrt{\mu_2}}\,l^3 = \frac{t\,(t^2 + 1)^{1.5}}{\sqrt{\mu_2}} \quad \text{...................(28)}$$

and, finally, since by Equation (17), $S_0 = N = \frac{a\sqrt{\pi}}{c\,d}\,e^{\frac{1}{4c^2}}$:

$$a = \frac{S_0 c\,d}{\sqrt{\pi}}\,e^{-\frac{1}{4c^2}} = \frac{S_0 c\,d}{\sqrt{\pi}}\,e^{-\log\sqrt{t^2+1}}$$

$$= \frac{N}{\sqrt{\pi}}\,\frac{1}{\sqrt{2 \log\,(t^2+1)}}\,\frac{t\,(t^2+1)^{1.5}}{\sqrt{\mu_2}}\,\frac{1}{\sqrt{t^2+1}}$$

$$= \frac{N}{\sqrt{2\,\pi\,\mu_2}}\,\frac{t\,(t^2+1)}{\sqrt{\log\,(t^2+1)}} \quad \text{...................(29)}$$

4.—The Duration Curve.—The probability that the variate lies between $-\,b$ (the zero of the function) and x is expressed by:

$$Y = a \int_{-b}^{x} e^{-c^2[\log d\,(x+b)]^2}\,dx \quad \text{............(30)}$$

or, making use of the transformation at the beginning of Article 2, Section II,

$$Y = \frac{a}{d} \int_{-\infty}^{\log d\,(x+b)} e^{z - c^2 z^2}\,dz$$

$$= \frac{a}{d}\,e^{\frac{1}{4c^2}} \int_{-\infty}^{\log d\,(x+b)} e^{-c^2\left(z - \frac{1}{2c^2}\right)^2}\,dz$$

Now, let $u = c\ (z - \frac{1}{2\ c^2})$, $du = c\ dz$, and the duration curve becomes:

$$Y = \frac{a}{c\ d} e^{\frac{1}{4c^2}} \int_{-\infty}^{c\left[\log d(x+b) - \frac{1}{2c^2}\right]} e^{-u^2} du$$

$$= \frac{N}{\sqrt{\pi}} \int_{-\infty}^{c \log \frac{d(x+b)}{t^2+1}} e^{-u^2} du \quad \ldots\ldots\ldots\ldots\ldots(31)$$

This is the ordinary probability integral with argument, $c \log \frac{d\ (x + b)}{t^2 + 1}$.

5.—The Elements of the Curve.—A few points on the curve are of special interest:

(*a*) The End Point: When $x = -b$, the curve obviously ends because there are no real values of y for $x < -b$. Since, for every real value of t there is one and only one real value of β and, conversely, it is a mere matter of convenience whether t or β is designated the coefficient of skewness. With this in mind, it is seen from Equation (27) that the skewness of the curve is a simple function of its end point, as was required of it in Article 1, Section II. In general, however, the end point of the curve does not correspond to the zero of the data. If one is quite sure of the zero, then this value may be taken for b, in terms of which, a, c, and d may be computed by Equations (26) to (29). When this is done the third moments of the curve and data do not quite agree, but it must be remembered that the probable error in the third moment is large when the distribution is not normal, and, therefore, agreement in the third and higher moments is not a good criterion of fit.

(*b*) The Mode: To obtain the mode,

$$\frac{dy}{dx} = -2\ a\ c^2\ \frac{\log d\ (x+b)}{x+b}\ e^{-c^2 [\log d\ (x+b)]^2} = 0 \quad \ldots(32)$$

For finite values of $x > -b$ this expression vanishes when the logarithmic factor vanishes; that is, when $d\ (x + b) = 1$. So that the mode is at:

$$x = \frac{1}{d} - b = b\left[\frac{1 - (t^2 + 1)^{1.5}}{(t^2 + 1)^{1.5}}\right] \ldots\ldots\ldots\ldots\ldots(33)$$

(*c*) The Median: To obtain the median, notice that,

$$\int_{-\infty}^{0} e^{-u^2} du = \int_{0}^{\infty} e^{-u^2} du$$

so that,

$$u = c \log \frac{d\,(x + b)}{t^2 + 1} = 0$$

or,

$$d\,(x + b) = t^2 + 1 \quad \text{.......(34)}$$

and,

$$x = \frac{\sqrt{\mu_2}}{t\sqrt{t^2 + 1}} - \frac{\sqrt{\mu_2}}{t} = b\left[\frac{1 - \sqrt{t^2 + 1}}{\sqrt{t^2 + 1}}\right] \quad \text{.......(35)}$$

Dividing Equation (35) by Equation (33):

$$\frac{\text{Median}}{\text{Mode}} = \frac{1 - (t^2 + 1)^{0.5}}{1 - (t^2 + 1)^{1.5}} \quad \text{.......(36)}$$

which, for all values of t, is less than 1. From Equation (36) it is seen that the median always lies between the mode and the mean.

6.—The Normal Curve as a Limiting Form.—The next step is to inquire what shape this curve will have as t approaches 0; that is, as the skewness vanishes:

$$\underset{t \to 0}{\text{limit}}\; a = \underset{t \to 0}{\text{limit}}\; \frac{N}{\sqrt{2\,\pi\,\mu_2}} \; \frac{t\,(t^2 + 1)}{\sqrt{\log\,(t^2 + 1)}}$$

$$= \underset{t \to 0}{\text{limit}}\; \frac{N}{\sqrt{2\,\pi\,\mu_2}} \; \frac{t\,(t^2 + 1)}{\sqrt{1 - \frac{t^2}{2} + \frac{t^4}{3} - \ldots}} = \frac{N}{\sqrt{2\,\pi\,\mu_2}} \quad \text{......(37)}$$

and,

$$\underset{t \to 0}{\text{limit}}\; c\,d = \underset{t \to 0}{\text{limit}}\; \frac{t\,(t^2 + 1)^{1.5}}{\sqrt{\mu_2}\;\sqrt{2 \log\,(t^2 + 1)}}$$

$$= \underset{t \to 0}{\text{limit}}\; \frac{t\,(t^2 + 1)^{1.5}}{\sqrt{2\,\mu_2}\; t \;\sqrt{1 - \frac{t^2}{2} + \ldots}} = \frac{1}{\sqrt{2\,\mu_2}} \quad \text{.........(38)}$$

so that, for small values of t, $cd = \dfrac{1}{\sqrt{2\,\mu_2}} + \epsilon$; and,

$$d = \frac{1}{c}\left(\frac{1}{\sqrt{2\,\mu_2}} + \epsilon\right) \quad \text{.......(39)}$$

in which, ϵ is a small quantity that vanishes with t.

Now, $b\,d = (t^2 + 1)^{1.5} = 1 + \dfrac{3}{2}\,t^2 + \ldots$ by the binomial theorem; or,

$$b\,d = 1 + 0\,(t^2) \quad \text{.......(40)}$$

in which, $0\,(t^2)$ is a small quantity that vanishes with t^2. Then:

$$\left.\begin{aligned} &\lim_{t\to 0} c \log d\,(x+b) = \lim_{t\to 0} \log\,[d\,(x+b)]^c \\ &= \lim_{c\to\infty} \log\left[\frac{x}{c}\left(\frac{1}{\sqrt{2\mu_2}}+\epsilon\right)+1+0\,(t^2)\right]^c \\ &= \log e^{\frac{x}{\sqrt{2\mu_2}}} = \frac{x}{\sqrt{2\mu_2}} \end{aligned}\right\} \quad \ldots\ldots (41)$$

The existence of this limit (Equation 41) may be shown by applying the rule of L'Hospital.[14] Combining these various results:

$$\lim_{\beta\to 0} a\, e^{-c^2 [\log d\,(x+b)]^2} = \frac{1}{\sqrt{2\pi\mu_2}}\, e^{-\frac{x^2}{2\mu_2}} \quad \ldots\ldots\ldots (42)$$

III.—The Use of the Function

1.—The Computation of the Parameters.—To fit the curve to a set of data compute the moments, μ_2 and μ_3, in the usual way; then compute $\beta = \dfrac{\mu_3}{\mu_2^{1.5}}$, followed by A, B, and t (Equation (25)). It will be found convenient to compute immediately the quantities, $t^2 + 1$; $t\,(t^2 + 1)$; $\sqrt{t^2 + 1}$; and $\sqrt{2.303 \ldots \log_{10}(t^2 + 1)}$. (The logarithms of the last section are to the base, e. Consequently, $\log x = 2.303 \ldots \log_{10} x$.)

In terms of these quantities the parameters are:

$$\sqrt{2\pi}\, a = \frac{N}{\sqrt{\mu_2}\cdot} \frac{t\,(t^2 + 1)}{\sqrt{2.303 \log_{10}(t^2 + 1)}}$$

$$b = \frac{\sqrt{\mu_2}}{t}$$

$$\sqrt{2}\, c = \frac{1}{\sqrt{2.303 \log_{10}(t^2 + 1)}}$$

and,

$$d = \frac{1}{\sqrt{\mu_2}}\, t\,(t^2 + 1)\sqrt{t^2 + 1}$$

Since Equation (8*a*) is the most usual form in which the normal curve is tabulated, the quantities, $\sqrt{2\pi}\, a$ and $\sqrt{2}\, c$, are computed instead of a and c.

Table 1 gives the values of these parameters for selected values of β for $\sigma = 1$. If σ is not 1, this table may be used with $\dfrac{x}{\sigma}$ and $\dfrac{a}{\sigma}$.

[14] "Advanced Calculus", by F. S. Woods, p. 18.

TABLE 1.—VALUES OF THE PARAMETERS FOR $\sigma = 1$

β	t	$\sqrt{2\pi}\,a$	b	$\sqrt{2c}$	d	β	t	$\sqrt{2\pi}\,a$	b	$\sqrt{2c}$	d
0.0	0	1.0000	......	......	......	2.2	0.643	1.5454	1.555	1.6996	1.0812
0.2	0.067	1.0029	14.925	14.9030	0.0674	2.4	0.690	1.6328	1.449	1.6021	1.2384
0.4	0.133	1.0180	7.518	7.5180	0.1366	2.6	0.735	1.7217	1.361	1.5223	1.4036
0.6	0.197	1.0438	5.076	5.1020	0.2085	2.8	0.777	1.8130	1.278	1.4550	1.5781
0.8	0.261	1.0861	3.831	3.8960	0.2881	3.0	0.818	1.9065	1.223	1.3976	1.7620
1.0	0.322	1.1310	3.106	3.2331	0.3733	3.2	0.857	2.0023	1.167	1.3479	1.9564
1.2	0.382	1.1858	2.621	2.7122	0.4679	3.4	0.895	2.1008	1.118	1.3041	2.1615
1.4	0.438	1.2466	2.281	2.4161	0.5707	3.6	0.931	2.2001	1.074	1.2656	2.3750
1.6	0.493	1.3145	2.027	2.1427	0.6841	3.8	0.966	2.3007	1.035	1.2315	2.5977
1.8	0.545	1.3865	1.834	1.9600	0.8057	4.0	1.000	2.4021	1.000	1.2012	2.8284
2.0	0.596	1.4652	1.678	1.8136	0.9406	...	.....	......	.....	......	......

Table 2 has been constructed to give deviations from the mean for selected frequencies (given herein as percentages of time), and is similar to tables[15] constructed by H. Alden Foster, M. Am. Soc. C. E., for Pearson's Type III curve. Only extreme values are given, as this curve plots fairly straight on logarithmic probability paper. In using such a table, however, it must be remembered that appearance, particularly of a graph plotted on logarithmic probability paper, is no test of the goodness of fit.[16]

TABLE 2.—SHORT TABLE OF EXTREME UPPER VALUES FOR $(CV) = 1$

Coefficient of skew $= \beta$	%-OF-TIME									
	90	10	1	0.1	0.01	0.001	0.0001	0.00001	0.000001	0.0000001
0.0.....	—1.28	+1.28	2.33	3.09	3.72	4.26	4.75	5.20	5.61	6.00
0.2.....	—1.25	+1.31	2.49	3.42	4.21	4.91	5.58	6.20	6.78	7.38
0.4.....	—1.23	+1.32	2.64	3.72	4.70	5.62	6.50	7.36	8.19	9.03
0.6.....	—1.20	+1.33	2.79	4.06	5.25	6.41	7.56	8.73	9.89	11.07
0.8.....	—1.16	+1.32	2.91	4.36	5.80	7.24	8.71	10.26	11.81	13.44
1.0.....	—1.12	+1.31	3.04	4.70	6.40	8.20	10.10	12.00	13.90	16.50
1.2.....	—1.09	+1.30	3.16	5.03	7.03	9.16	11.48	14.03	16.74	19.74
1.4.....	—1.05	+1.29	3.25	5.32	7.60	10.19	13.00	16.15	19.22	23.60
1.6.....	—1.03	+1.28	3.36	5.66	8.29	11.25	14.66	18.56	22.89	27.87
1.8.....	—1.00	+1.26	3.45	5.96	8.91	12.32	16.34	21.02	26.34	32.50
2.0.....	—0.97	+1.24	3.53	6.24	9.54	13.43	18.11	23.66	29.99	37.78
2.2.....	—0.94	+1.22	3.60	6.50	10.12	14.49	19.85	26.34	33.91	43.08
2.4.....	—0.91	+1.20	3.65	6.76	10.71	15.58	21.67	29.30	38.03	49.21
2.6.....	—0.89	+1.18	3.71	6.99	11.28	16.64	23.48	31.84	42.57	55.04
2.8.....	—0.87	+1.16	3.75	7.21	11.82	17.70	25.18	34.80	46.94	61.26
3.0.....	—0.84	+1.14	3.79	7.42	12.34	18.77	27.15	37.85	51.00	67.78
3.2.....	—0.82	+1.12	3.83	7.61	12.83	19.72	28.77	40.89	55.60	74.70
3.4.....	—0.81	+1.11	3.86	7.78	13.33	20.80	30.61	43.91	60.27	81.75
3.6.....	—0.79	+1.09	3.88	7.95	13.78	21.78	32.35	46.83	64.90	88.87
3.8.....	—0.77	+1.07	3.90	8.11	14.21	22.64	34.28	49.58	70.08	95.92
4.0.....	—0.76	+1.05	3.92	8.26	14.69	23.61	35.72	52.69	74.44	103.93

A point that will be found useful in plotting the curve is the mode, $x = \frac{1}{d} - b$. If there is sufficient skewness, the end point, $x = -b$, will also help.

2.—Examples.—

Example *a.*—A problem worked out in full detail will illustrate the application of these formulas. The first problem analyzed is that given by

[15] "Theoretical Frequency Curves and Their Application to Engineering Problems", by H. Alden Foster, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 142.

[16] "Probability and Its Engineering Uses", by Thornton C. Fry, Chapter IX, N. Y., Van Nostrand Co., 1928.

Elderton,[17] which is based upon a set of data to which he fits a transition Type III curve. This particular problem was chosen because it is one in which Pearson's curve breaks up into an L-shape. Furthermore, Pearson's Type III curve may be compared fairly with the writer's in that both are specified by the same parameters. The data are listed in Table 3.

TABLE 3.—DATA UPON WHICH EXAMPLE a IS BASED ($N = 251$).

x (1)	Observed frequency (2)	Frequency from Pearson's Type III curve (3)	Mid-ordinates of writer's curve (4)
0.400	...	...	0
1	44	59	62
1.575	...	...	125
2	134	111	109
3	45	45	49
4	12	20	19
5	8	9	8
6	3	4	3
7	1	2	2
8	3	1	1

Although the frequencies of Column (3), Table 3, are quite close to the observed frequencies, the mid-ordinates of Pearson's curve show it to be a poor representative of this distribution, as may be seen by referring to the illustration in Elderton's book.[17] Elderton gives the elements: The mean at $x = 2.335$; $\mu_2 = 1.442$ ($\sqrt{\mu_2} = 1.201$; $\mu_2^{1.5} = 1.730$); and, $\mu_3 = 3.607$. Consequently, $\beta = \dfrac{3.607}{1.730} = 2.085$ and, by Equation (25):

$$A = 1.042 + \sqrt{2.022} = 1.042 + 1.421 = 2.463$$

$$B = 1.042 - 1.421 = -0.379$$

and,

$$t = 1.345 - 0.724 = 0.621$$

Of course, this value of t could have been obtained closely enough from Table 1.

Next, compute the quantities: $t^2 + 1 = 1.345$; $t\,(t^2 + 1) = 0.861$; $\sqrt{t^2 + 1} = 1.178$; and, $\sqrt{2.303 \ldots \log (t^2 + 1)} = 0.574$. With these values, the parameters of the curve may be computed, as follows:

$$\sqrt{2\pi}\, a = \frac{251 \times 0.861}{1.201 \times 0.574} = 313$$

$$b = \frac{1.201}{0.621} = 1.934$$

$$\sqrt{2}\, c = \frac{1}{0.574} = 1.741$$

[17] "Frequency Curves and Correlation", by W. P. Elderton, p. 90, 1927.

and,

$$d = \frac{0.861 \times 1.178}{1.201} = 0.844$$

The ordinates of the curve are computed by constructing Table 4, each column being constructed from preceding columns. The values in Column

TABLE 4.—COMPUTATION TABLE FOR DETERMINING ORDINATES OF FREQUENCY CURVE

$x = X - M$ (1)	$x + b$ (2)	$d(x + b)$ (3)	log (Column (3)) (4)	$2.303 \times \sqrt{2} \times c$(Column (4)) (5)	ϕ_0 (Column (5)) (6)	$\sqrt{2\pi}\, a \times$ (Column (6)) (7)
−1.335	0.6	0.506	−0.296	−1.188	0.199	62
−0.335	1.6	1.350	0.130	0.522	0.348	109
+0.665	2.6	2.195	0.341	1.370	0.156	48.8
1.665	3.6	3.040	0.483	1.940	0.061	19.1
2.665	4.6	3.881	0.589	2.365	0.025	7.8
3.665	5.6	4.725	0.674	2.705	0.010	3.1
4.665	6.6	5.570	0.746	2.995	0.005	1.6
5.665	7.6	6.410	0.807	3.240	0.002	0.6
6.665	8.6	7.252	0.860	3.450	0.001	0.3

(6) are from tables of Equation (8a) compiled by Messrs. F. C. Mills and D. H. Davenport.[18] The mode is at $x = \dfrac{1}{d} - b = -0.75$; $X = 2.335 - 0.75 = 1.575$; and, $y = 313 \times 0.3989 = 125$. The end point is at $x \doteq -1.934$; and $X = 2.335 - 1.934 = 0.4$.

Example b.—The next problem is one given by Fisher,[19] who makes the statement that the statistical assistants working on the problem were unable to fit the data by means of Pearson's curves.

Fisher uses the logarithmically transformed function, (Equation (10)), as a generic function. After calculating the values of m and n for this particular problem, he then determines the constants, c_0, c_3, and c_4, by the method of least squares. Unfortunately, he does not give his computed ordinates; nor does he give the ordinates for the distribution,[20] so that his curve and that of the writer for this problem can be compared only by sight from slightly different perspectives. However, a glance at Fisher's diagram[20] and at Fig. 1 of this paper is sufficient to show that the latter is at least as good a fit. Furthermore, the work required to fit these data by the present method is quite negligible compared to that which Fisher must do to obtain a result that is "satisfactory for all practical purposes".

The observed data and the ordinates computed by the present method are given in Table 5. The elements of this distribution are the following: The mean is at $x = 5.830$, with $\mu_2 = 7.245$ ($\sqrt{\mu_2} = 2.695$; $\mu_2^{1.5} = 19.502$);

18 "Problems and Tables in Statistics", by F. C. Mills and D. H. Davenport, Henry Holt & Co., 1925.

19 "The Mathematical Theory of Probability", by Arne Fisher, 1930, p. 258.

20 *Loc. cit.*, Fig. 4.

FIG. 1

$\beta = 1.405$; $t = 0.499$; $t^2 + 1 = 1.248$; $t\,(t^2 + 1) = 0.623$; $\sqrt{t^2 + 1} = 1.120$; $\sqrt{2.303 \log (t^2 + 1)} = 0.472$; $\sqrt{2\,\pi}\,a = 9\,440$; $b = 5.41$; $\sqrt{2}\,c = 2.12$; and, $d = 0.259$. The mode is at $x = 4.28$ and $y \doteq 3\,765$.

TABLE 5.—OBSERVED DATA AND ORDINATES

X	Observed frequencies	Computed mid-ordinates	X	Observed frequencies	Computed mid-ordinates
0.42	1	0	12	272	249
1	9	1	13	186	161
2	745	619	14	141	107
3	2 264	2 625	15	110	71
4	3 828	3 719	16	72	47
5	3 801	3 525	17	43	33
6	2 711	2 780	18	17	17
7	1 918	1 921	19	14	14
8	1 339	1 355	20	3	10
9	884	904	21	2	8
10	533	586	22	...	5
11	380	382	23	...	2

Example *c*.—For this problem the writer has taken a 49-yr rainfall record of Chapel Hill, N. C. In order of magnitude the yearly rainfalls are as shown in Table 6. The elements are: $\beta = 0.522$; $\sqrt{\mu_2} = 7.49$; the mean $= 47.72$; $t^2 + 1 = 1.032$; $\sqrt{2\,\pi}\,a = 6.17$; $b = 42.1$; $\sqrt{2}\,c = 5.14$; and $d = 0.025$.

TABLE 6.—YEARLY RAINFALL, IN INCHES, IN ORDER OF MAGNITUDE

31.88	40.73	46.86	50.39	54.06
32.17	40.82	47.31	50.82	54.54
37.99	40.83	47.37	51.07	54.87
38.47	42.00	47.84	51.38	55.92
38.58	42.41	48.52	52.14	56.85
39.19	43.34	48.56	52.21	58.19
39.42	43.62	48.87	52.81	61.34
39.75	44.81	49.06	53.17	62.25
40.22	45.20	49.42	53.30	67.15
40.45	46.55	50.26	53.57	.....

This record has been plotted cumulatively according to the usual convention[21] (against 0.5, 1.5, 2.5, etc.), the circles in Fig. 2 representing these points. The ordinates for the smooth curve have been computed according to the data in Table 7.

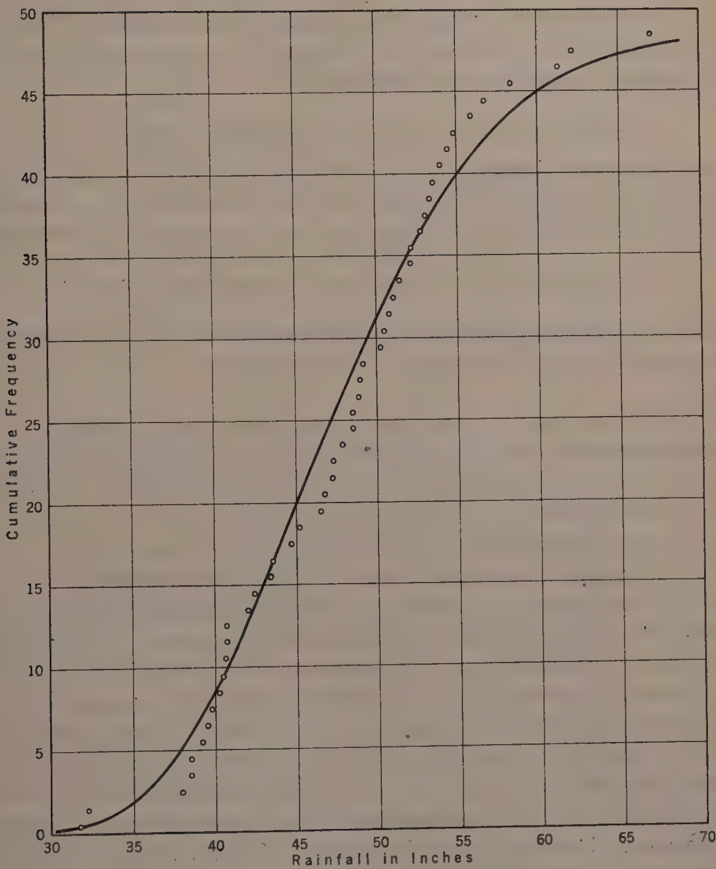

FIG. 2

TABLE 7.—COMPUTATION OF ORDINATES IN FIG. 2

X (1)	$x = X - M$ (2)	$x + b$ (3)	$d\,(x + b)$ (4)	$\frac{d\,(x + b)}{t^2 + 1}$ (5)	log (Column (5)) (6)	11.81 (Column (6)) (7)	Y (8)	$N\,Y$ (9)
30....	—17.72	24.38	0.608	0.589	—0.230	—2.72	0.003	0.15
35....	—12.72	29.38	0.734	0.711	—0.148	—1.75	0.030	1.47
40....	— 7.72	34.38	0.858	0.831	—0.080	—0.95	0.171	8.38
45....	— 2.72	39.38	0.984	0.952	—0.021	—0.25	0.401	19.65
50....	2.28	44.38	1.109	1.073	0.031	0.37	0.644	31.53
55....	7.28	49.38	1.234	1.195	0.077	0.92	0.821	40.25
60....	12.28	54.38	1.359	1.316	0.119	1.41	0.921	45.15
65....	17.28	59.38	1.485	1.439	0.158	1.87	0.969	47.49

[21] "Duration Curves", by H. Alden Foster, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 99 (1934) p. 1213.

Column (7), Table 7, gives 2.303 $\sqrt{2}\, c \log \dfrac{d\,(x + b)}{t^2 + 1}$ for the corresponding values of x. To obtain the values in Column (8) it is necessary to select the values of the probability integral (as given by Mills and Davenport,[14] for instance) corresponding to the values in Column (7). Then, 0.5 + (Column (7)) = (Column (8)), because the tables are calculated so as to give the area from 0 up to the ordinate, x.

IV.—The Most General Frequency Function

1.—The Range of Variation.—As has been pointed out in Section III, instead of computing the third moment of the distribution, one may determine, somehow, the lower limit of the variation; then, through the relation, $b = \dfrac{\sigma}{t}$, the constants of the curve can be computed. The two processes will not lead, in general, to precisely the same result, but the discrepancy should come well within the margin of probable errors, if the present theory is correct.

It is not always easy to determine this lower point, however. In considering the heights of a group of men, for instance, one would not be justified in taking zero as the lower limit of the variation; 1 ft or 2 ft would be closer to the actual limit. Even if one were to take 0 as this lower limit, however, one would be doing something more rational than taking $-\infty$, as is always done when the normal curve is used in connection with such statistics. In fact, even in the most symmetrical cases ordinarily considered, the normal curve gives the poorest estimate of the range of the variation.

Unlimited variations do occur: There is a finite chance that a run of n heads will occur in tossing a coin no matter how large n may be; but in physical fluctuations these chances do not usually occur because the phenomenon in question does not exist in these extreme aspects.

Consequently, the problem at hand is to determine the range of variation of a statistical series under consideration. For this determination there are two alternatives: The range may be determined statistically (which is, in effect, Pearson's procedure), or, it may be determined from purely physical considerations. The statistical determination is a purely formal process which leads close to the true answer only when the data are abundant, but quite a fair estimate may be made from non-statistical considerations of the range of fluctuation of a great many physical phenomena. For instance, a stream, will certainly not run less than dry, and the greatest possible flood that can occur on it is certainly some function of its drainage area, geographical and geological location, etc.

In what follows it will be assumed that this estimate may be made. Let b = the maximum lower fluctuation from the mean of the statistics, and g = the maximum upper fluctuation.

2.—The Totally Bounded Function.—The construction of a function with both lower and upper limits is a simple generalization of the function already

discussed; but the analysis of it is not straightforward. The preceding analysis, however, places one in a position to determine the parameters of the totally bounded function in terms of its end points. Since it is no longer a question of determining these parameters by the method of moments (however desirable such a determination may be), the integral curve will be dealt with directly.

The probability integral will be taken in the form in which it is most usually tabulated, namely,

$$Y=\frac{N}{\sqrt{\pi}}\int_{-\infty}^{z} e^{-\frac{u^2}{2}}\,du \quad \ldots\ldots\ldots\ldots\ldots\ldots (43)$$

For the partly bounded function, by substituting $\frac{\sigma}{b}$ for t in the preceding formulas:

$$z = c \log d\,(x+b) \ldots\ldots\ldots\ldots\ldots\ldots (44a)$$

in which,

$$d = \frac{\sigma}{b^2}\sqrt{b^2+\sigma^2} \ldots\ldots\ldots\ldots\ldots\ldots (44b)$$

and,

$$\frac{1}{c} = \sqrt{\log \frac{\sigma^2}{b^2}\,(b^2+\sigma^2)} \ldots\ldots\ldots\ldots\ldots\ldots (44c)$$

From Equations (43) and (44), by an obvious extension, the following set of formulas is obtained for the totally bounded curve:

$$z = pc \log d\left(\frac{x+b}{g-x}\right) \ldots\ldots\ldots\ldots\ldots\ldots (45a)$$

in which,

$$d = \frac{g^2}{b^2}\sqrt{\frac{\sigma^2+b^2}{\sigma^2+g^2}} \ldots\ldots\ldots\ldots\ldots\ldots (45b)$$

$$\frac{1}{c} = \sqrt{\log \frac{g^2}{b^2}\left(\frac{\sigma^2+b^2}{\sigma^2+g^2}\right)} \ldots\ldots\ldots\ldots\ldots\ldots (45c)$$

and,

$$p = \sqrt{\frac{g-b}{g+b+2\,\sigma}} \ldots\ldots\ldots\ldots\ldots\ldots (45d)$$

The factor, p, has been introduced to make the second moment of the curve agree with that of the statistics. The determination has been empirical and, consequently, this factor may be found to require further modification. The considerations that have guided the writer in the present determination are the following: p must approach 1 as g approaches ∞, and -1 as b approaches ∞. Furthermore, the product, pc, must remain finite and determinate as g approaches b.

With these elements the curve represents the most general homograde probability function, according to the present theory. It begins at a distance, b, below the mean and ends at a distance, g, above it, these points coinciding with the absolute limits of the statistics, and its mean and standard deviation coincides with those of the observed distribution. By inspection, it is seen that the partly bounded function discussed in Sections II and III is obtained by permitting either g or b recede to ∞, the other remaining finite (in one case, the curve is right-skewed; in the other, left-skewed).

The form of this function must now be determined as g becomes equal to b; that is, as the curve becomes symmetrical, but bounded. When $g = b$, obviously $d = 1$. Assume that g is nearly equal to b so that $g^2 - b^2$ is small; then,

$$\log \frac{g^2}{b^2}\left(\frac{\sigma^2 + b^2}{\sigma^2 + g^2}\right) = \log\left[1 + \frac{\sigma^2}{b^2}\left(\frac{g^2 - b^2}{g^2 + \sigma^2}\right)\right]$$

$$= \frac{\sigma^2}{b^2}\left(\frac{g^2 - b^2}{g^2 + \sigma^2}\right) + O\left([g^2 - b^2]^2\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (46)$$

Consequently,

$$p\,c = \sqrt{\frac{g - b}{g + b + 2\,\sigma}\ \frac{\sigma^2}{b^2}\ \frac{g^2 + \sigma^2}{g^2 - b^2}} + O\left(g^2 - b^2\right) \quad \ldots\ldots\ldots (47)$$

and,

$$\lim_{g \to b} p\,c = \frac{b}{2\,\sigma}\sqrt{\frac{b^2 + \sigma^2}{b^2 + \sigma\,b}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (48)$$

Therefore, for symmetrical, bounded fluctuations the argument becomes,

$$z = \frac{b}{2\,\sigma}\sqrt{\frac{b^2 + \sigma^2}{b^2 + \sigma\,b}}\log\left(\frac{x + b}{b - x}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (49)$$

from which the normal curve is obtained by letting b recede to ∞, when one obtains, simply,

$$z = \frac{x}{\sigma} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (50)$$

It would be highly desirable to determine the parameters of this general function in terms of the moments (for this, a fourth moment would be

TABLE 8.—THE PROBABILITY INTEGRAL

Probability Y, in %-of-time	Parameter, z	Probability Y, in %-of-time	Parameter, z	Probability Y, in %-of-time	Parameter, z
0.0000001	6.00	0.1	3.09	50	0.00
0.000001	5.61	1	2.33	60	−0.25
0.00001	5.20	10	1.28	70	−0.52
0.0001	4.75	20	0.84	80	−0.84
0.001	4.26	30	0.52	85	−1.04
0.01	3.72	40	0.25	90	−1.28
				95	−1.64

required since the condition of termination at the upper end has been added) because, whether or not the present theory is tenable, this function represents the most flexible and stable curve of any in use at present. The writer, however, has found unsurmountable mathematical difficulties on the road to this goal. Table 8 is a short solution of the probability integral.

Example *d*.—To illustrate the use of the most general function given by Equations (45) the distribution of the flow peaks of the Tennessee River, at Chattanooga Tenn., will be considered. In the 57-yr daily flow record of the Chattanooga Station there are 2 440 peaks, ranging from a minimum of 3 360 cu ft per sec, to a maximum of 361 000 cu ft per sec. Their arithmetic average was found to be 49 550 cu ft per sec.

These peaks were grouped into classes of intervals of 5 000 cu ft per sec, so that $X = 1$ is the mid-ordinate (in class units) of the 0—5 000 class, or 2 500; $X = 2$ is the mid-ordinate of the 5 000—10 000 class, or 7 500; etc. The zero flow is at $X = 0.5$, and the mean is $M = 10.41$. The limits, b and g, were taken as 10 and 120 class units, respectively. This makes the range of fluctuation approximately from 0 to 650 000 cu ft per sec. No justification is offered for the selection of these limits. They are used merely for illustration. The standard deviation was computed to be 9.459 class units.

Substitution in Equations (45*b*), (45*c*), and (45*d*), gives $d = 16.47$, $c = 1.257$, and $p = 0.86$. In Table 9 three ordinates are solved to illustrate

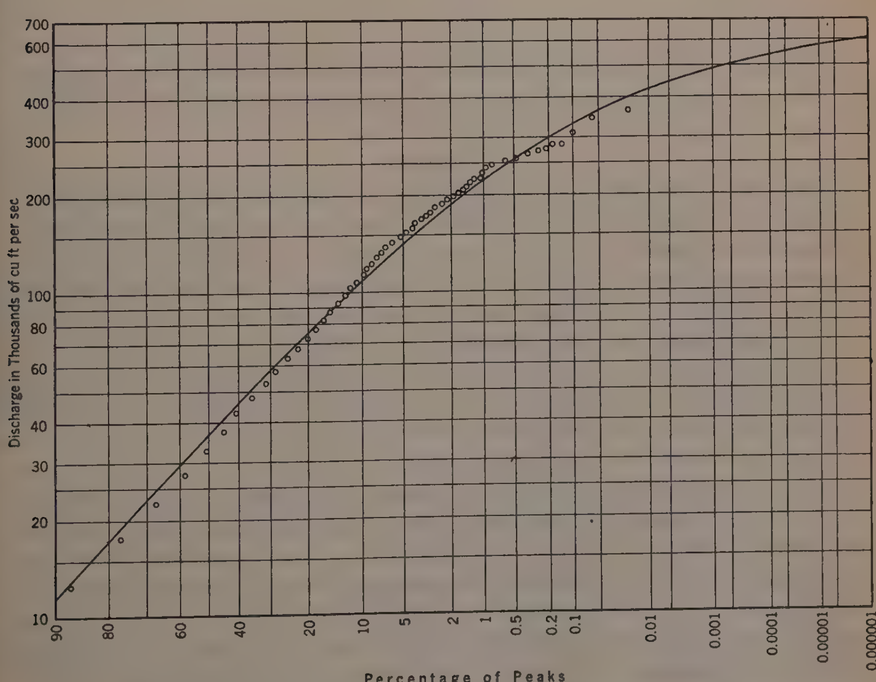

FIG. 3.—FLOW PEAKS, TENNESSEE RIVER AT CHATTANOOGA, TENN., 1875–1931.

the computation. Fig. 3 shows the observed cumulative frequencies[21] compared with the theoretical curve plotted on extreme value logarithmic probability paper.

TABLE 9.—COMPUTATION OF PROBABILITY

X (1)	$x = X - M$ (2)	$x + b$ (3)	$g - x$ (4)	$\frac{x+b}{g-x}$ (5)	$d\left(\frac{x+6}{g-x}\right)$ (6)	log (6) (7)	$\mu c \times$ (7) (8)	ϕ_o (9)	Probability Y, in % of-time (10)
5....	−5.410	4.59	125.41	0.0366	0.6027	−0.506	−0.548	0.29116	70.88
10....	−0.41	9.59	120.41	0.0796	1.3107	+0.2708	+0.293	0.61409	38.59
90....	79.59	89.59	40.41	2.2170	36.5060	3.597	3.89	0.99995	0.005

Column (9), Table 9, was obtained from the tables of Mills and Davenport.[18] If only a few values on the curve are required, Table 8 may be used as follows: Suppose it is required to find the flood likely to occur 0.01% of the time (assuming the floods equally spaced in time). Opposite 0.01 in Table 8, the value, 3.72, of z is found. The corresponding value of x may now be found by Equation (45a), arranging the computation as follows:

Y....................	0.01	$gw - bd$............	3 585.3
z....................	3.72	$w + d$............	47.72
$v = \frac{z}{pc}$..............	3.442	$\frac{gw - bd}{w + d} = x$........	75.1
$w = e^v$..............	31.25		

The last item, $x = 75.1$, is the deviation in class units from the mean. In cubic feet per seconds this flood is, $75.1 \times 5\,000 + 49\,550 = 425\,050$.

CONCLUSION

Equations (45) were constructed on the assumption that the most general homograde frequency function is characterized by its standard deviation and the maximum possible lower and upper fluctuations from its mean; that the normal is the limiting form of this function when the range is unlimited in both directions; and that the general shape of the normal is always preserved as closely as possible by this most general curve. If the curve becomes unlimited on one side, the other side remaining limited, the partly bounded function of Equation (11) is obtained, the analysis of which has suggested the form of the general function. If the two limits are equal, the symmetrical curve of Equation (49), is obtained.

As in the case of the normal, the errors of sampling of this distribution are those introduced in determining the mean and the standard deviation, the errors introduced in the determination of the end points being non-statistical in nature. Although this does not do away with the difficulties encountered in analyzing skewed frequency distributions, it does shift part of the responsibility from the shoulders of the statistician to those of the engineer or physicist, who may set limits to the problem from extra-statistical considerations.

Acknowledgment

For valuable criticism and suggestions, as well as for having introduced him to a fertile and wide field of analysis, the writer wishes to take this opportunity of expressing his indebtedness to Thorndike Saville, M. Am. Soc. C. E.

While employed as Associate Statistical Engineer by the United States Geological Survey on the work of the Mississippi Valley Committee, the writer has had an opportunity to develop the totally bounded function described in this paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

ANALYSIS OF CONTINUOUS STRUCTURES BY TRAVERSING THE ELASTIC CURVES

BY RALPH W. STEWART,[1] M. AM. SOC. C. E.

SYNOPSIS

A method of analyzing the moments in the members of continuous frames by a geometrical solution of the alignment of their elastic curves, is presented in this paper. Memorized or copied slope-deflection equations are not used, and a series of rules for the signs of moments, rotations, and deflections is unnecessary.

PRINCIPLES INVOLVED

Three basic principles are involved[2]:

(1) The angular change of the tangents at any two points on the elastic curve of a flexed beam is equal to the area between the two corresponding sections on the $\frac{M}{EI}$ - diagram.

(2) The curvature mentioned in Principle (1) may be represented as an angle which, in radians, is numerically equal to the corresponding $\frac{M}{EI}$ - area. This angle is platted opposite the center of gravity of the part of the $\frac{M}{EI}$ - diagram under consideration.

(3) For any unit of the $\frac{M}{EI}$ - diagram a triangular traverse of the corresponding unit of the elastic curve can be constructed. This triangle is composed of the tangents and the chord of the elastic curve. The angle between the tangents is given by Principle (2). Each angle between a tangent and the chord is directly proportional to the opposite side. This is because in beam flexure the angles are so small that they may be taken as equal to their sines.

NOTE.—Discussion on this paper will be closed in January, 1935, *Proceedings*.

[1] Engr. of Bridge Design, Los Angeles, Calif.

[2] *Civil Engineering*, February, 1934, p. 88.

The traversing triangle thus constructed can be solved if one angle and one side are known.

Applications to Structures

In the cantilever beam (Fig. 1), the traversing triangle is ABC. The angle, Δ, is equal to the area of the moment diagram $= \frac{M\,l}{2\,EI}$. Angle $CAB = \frac{2}{3}\,\Delta$ and Angle $CBA = \frac{1}{3}\,\Delta$. The deflection equals:

$$d = \Delta \times \frac{2}{3}\,l = \frac{M\,l^2}{3\,EI} = \frac{P\,l^3}{3\,EI} \qquad (1)$$

As an alternate solution the full length of the beam may be multiplied by $\frac{2}{3}\,\Delta$ as shown by Fig. 1(*b*).

Fig. 2 represents a span of a continuous beam in flexure which involves end slopes, end moments, and end translation as indicated. In dividing the

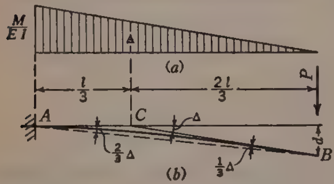

Fig. 1.

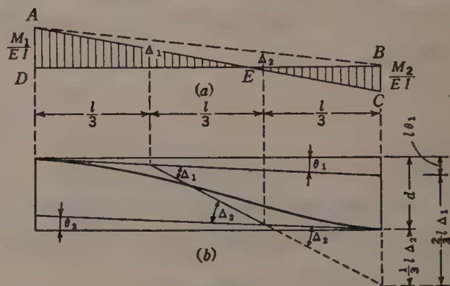

Fig. 2.

moment diagram into units it will simplify the solution to add the dotted line, AB, and treat the moment diagram as being composed of two triangles, ABD and BAC, each running the full length of the beam. The triangle, ABE, is a positive addition to the real M_1-triangle and a negative addition to the real M_2-triangle and, therefore, cancels its effect on the final deflection and end slope of the span.

From the center of gravity of Triangle ABD drop a vertical to the tangent through the left end of the elastic curve and lay off Δ_1 equal to the area of Triangle ABD. From the center of gravity of Triangle BAC drop a vertical to the lower leg of Δ_1 and lay off Δ_2 equal to Area BAC. Produce the upper leg of Δ_2 to the right end of the beam and draw the horizontal line indicating the end slope, θ_2.

From Fig. 2(*b*), it is now possible to read the following equations:

$$\theta_1 + \Delta_1 = \theta_2 + \Delta_2 \qquad (2)$$

and,

$$d = l\,\theta_1 + \frac{2}{3}\,\Delta_1\,l - \frac{1}{3}\,\Delta_2\,l \qquad (3)$$

Solving:

$$-\Delta_1 = 2\,\theta_1 + \theta_2 - \frac{3\,d}{l} \quad \text{.....................(4)}$$

Equating Δ_1 to its value, $\frac{M_1\,l}{2\,EI}$, and solving for M_1:

$$-M_1 = 2\,E\,\frac{I}{l}\left(2\,\theta_1 + \theta_2 - 3\,\frac{d}{l}\right) \quad \text{..............(5)}$$

Equation (5) will be recognized as a standard slope-deflection equation. It will not be used in subsequent solutions, but is introduced to illustrate the utility of the traverse method in deriving it. The line, $\Delta_1\,\Delta_2$ (Fig. 2(b)), is not tangent to the elastic curve, but crosses it at an angle, due to the addition of Triangle ABE to the moment diagram. If the true, shaded, moment triangles were used for locating and evaluating the Δ's, the line, $\Delta_1\,\Delta_2$, would be tangent to the elastic curve at its point of contraflexure, but the distances from the ends of the beam to the Δ's would be inconvenient and would make the solution complicated.

In order to illustrate the solution of the moments in a continuous frame by the elastic curve traverse without encumbering the problem with avoidable complications, reference is made to Fig. 3. This diagram shows a symmetrical frame with fixed base columns carrying a concentrated load at the center. The columns and the beams have equal lengths and cross-sections.

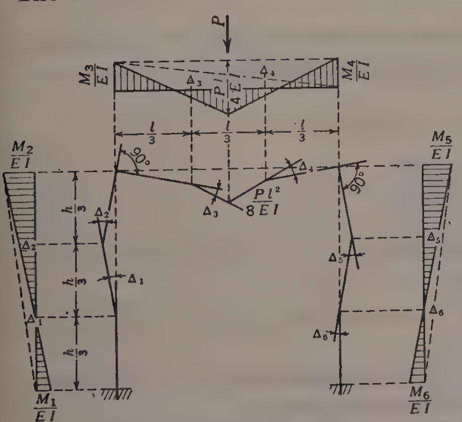

FIG. 3.

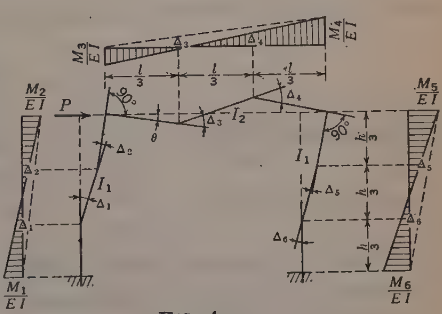

FIG. 4.

The true moment triangles in the beam are shown by the shaded areas. In order to simplify the traverse, the unshaded trapezoidal area between the closing line and the top of the diagram is added to the shaded center $\frac{M}{EI}$-triangle, thereby forming a "simple-beam" $\frac{M}{EI}$-triangle, the middle ordinate of which is $\frac{P\,l}{4\,EI}$. The combined area of the shaded end $\frac{M}{EI}$-triangles is also increased by the area of this same trapezoid. The resultant

curvature in the beam will then be the same as if the true shaded moments only were used, and the diagram may be treated as composed of three $\frac{M}{EI}$ - triangles, comprising the end $\frac{M}{EI}$ - triangles the base of which runs the full length of the beam, and the simple-beam $\frac{M}{EI}$ - triangle the height and area of which are $\frac{P\,l}{4\,EI}$ and $\frac{P\,l^2}{8\,EI}$, respectively. This device is used and discussed in treatises on "slope deflection", to which reference may be made, if further study is desired.[8]

The deflection curves may now be traversed. Each angle in the traverse is equal to the area of its corresponding $\frac{M}{EI}$ - triangle. Beginning at the bottom of the left column the traverse is as follows (the signs for the angles needing no discussion as they are the same as would be used for azimuth in a land survey):

$$-\Delta_1 + \Delta_2 + 90° + \Delta_3 - \frac{P\,l^2}{8\,EI} + \Delta_4 + 90° + \Delta_5 - \Delta_6 = 180° \quad \text{..(6)}$$

in which, Δ_2, Δ_3, Δ_4, and Δ_5 are all equal since their values of $\frac{M}{EI}$ and the lengths of members are equal. There being no side sway the triangles along the upper two-thirds of the columns are equilateral; consequently, $\Delta_2 = 2\,\Delta_1$ and $\Delta_5 = 2\,\Delta_6$. Therefore, Equation (6) reduces to $3\,\Delta_2 = \frac{P\,l^2}{8\,EI}$, or,

$$\Delta_2 = \frac{P\,l^2}{24\,EI} \quad \text{..........................(7)}$$

Substituting $\frac{M_2\,l}{2\,EI}$ for Δ_2, the corner moments are $\frac{P\,l}{12}$. The moments at the bases of the columns are $\frac{P\,l}{24}$, and the center moment is $\frac{P\,l}{4} - \frac{P\,l}{12} = \frac{P\,l}{6}$. As shown by Equation (6) the signs of the corner moments are opposite the signs of the center moment and column base moments.

The joint rotations do not enter into the solution, but the equations are formed from the curvature units in the members. This distinguishes the traverse method from the slope-deflection method in which standard equations involving joint rotations are applied.

Fig. 4 illustrates the application of the traverse method to an unsymmetrical frame having fixed base columns and subjected to a horizontal force, P. The material in the frame is assumed to be uniform and E may be eliminated from the equations.

[8] *Bulletin 108*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.; Concrete Engineers Handbook, by Hool and Johnson; etc.

From the $\frac{M}{I}$-diagram, $\Delta_2 = \frac{M_2\, h}{2\, I_1}$ and $\Delta_3 = \frac{M_3\, l}{2\, I_3}$; but, since $M_2 = M_3$, $\Delta_2 = \frac{I_3\, h}{I_1\, l}\ \Delta_3 = k_1\ \Delta_3$; similarly, $\Delta_5 = k_2\ \Delta_4$. The angles and courses in the deflected frame constitute a traverse for which the following equations may be written: $\Delta_1 - \Delta_2 + 90° - \Delta_3 + \Delta_4 + 90° + \Delta_5 - \Delta_6 = 180°$; or,

$$\Delta_1 - \Delta_2 - \Delta_3 + \Delta_4 + \Delta_5 - \Delta_6 = 0 \qquad (8)$$

The vertical deflection of the right end of the beam with reference to the left end is zero; or,

$$l\ \theta - \frac{2}{3}\ l\ \Delta_3 + \frac{1}{3}\ l\ \Delta_4 = 0 \qquad (9)$$

Since, $\theta = \Delta_1 - \Delta_2$, Equation (9) becomes:

$$\Delta_1 - \Delta_2 - \frac{2}{3}\ \Delta_3 + \frac{1}{3}\ \Delta_4 = 0 \qquad (10)$$

Similarly,

$$\Delta_6 - \Delta_5 - \frac{2}{3}\ \Delta_4 + \frac{1}{3}\ \Delta_3 = 0 \qquad (11)$$

Since the lateral deflections of the column tops are equal,

$$\frac{2}{3}\ \Delta_1\, h - \frac{1}{3}\ \Delta_2\, h = \frac{2}{3}\ \Delta_6\, h - \frac{1}{3}\ \Delta_5\, h$$

from which,

$$2\ \Delta_1 - \Delta_2 = 2\ \Delta_6 - \Delta_5 \qquad (12)$$

Since the sum of all column moments $= Ph$,

$$M_1 + M_2 + M_5 + M_6 = Ph$$

from which,

$$\Delta_1\, I_1 + \Delta_2\, I_1 + \Delta_5\, I_2 + \Delta_6\, I_2 = \frac{P\, h^2}{2} \qquad (13)$$

Substituting $\frac{\Delta_2}{k_1}$ for Δ_3 and $\frac{\Delta_5}{k_2}$ for Δ_4, in Equations (8) to (12), solving for Δ_2, and then substituting for Δ_2 its value, $\frac{M_2\, h}{2\, I_1}$:

$$M_2 = \frac{P\, h}{2} \times \frac{1}{1 + \frac{1}{3\, k_1} - \frac{1}{6\, k_2} + q\left(1 + \frac{1}{3\, k_2} - \frac{1}{6\, k_1}\right)} \qquad (14)$$

in which, $q = \frac{2 + k_1}{2 + k_2}$. If $I_1 = I_2$, then $k_1 = k_2$ and $q = 1$, and Equation (14) reduces to a much simpler form.

In practical problems of this type the work is greatly simplified, as the numerical values of constants, such as k_1, k_2, I_1, I_2, and $\frac{P h^2}{2}$, are inserted in the initial equations.

The use of the traverse method for the variable moment of inertia and a settled support is illustrated by Fig. 5, which represents a beam fixed at

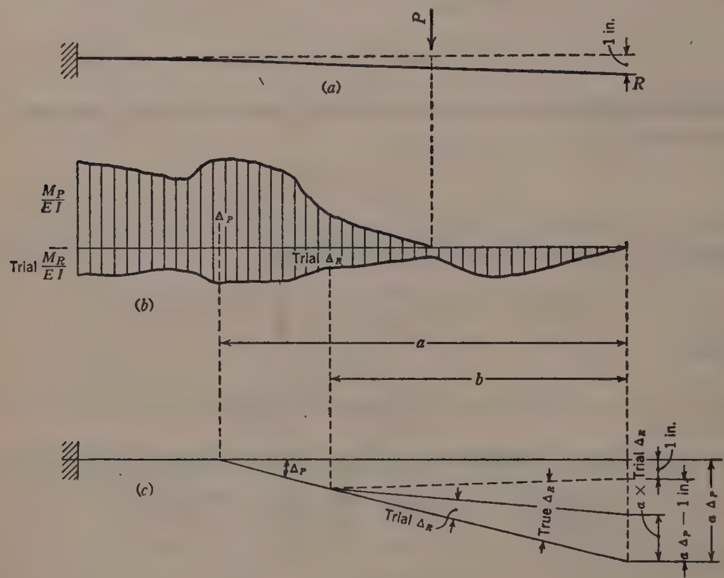

FIG. 5.

one end and with a support which has settled 1 in. at the other. The moment of inertia and the modulus of elasticity may both be variable. From Maxwell's theorem, the final distortion of the beam from the combined action of the external forces may be treated as the sum of the distortions, due to each force acting separately.

With the force, P, acting alone, the deflection of the right end of the beam would be $a\,\Delta_P$ (Fig. 5(c)), in which, Δ_P is the area of the $\frac{M}{EI}$ - diagram for the beam considered as a cantilever supporting the load, P, and a is the distance from the right end to the center of gravity of this $\frac{M}{EI}$ - area. Assume a trial reaction, R, compute the appurtenant $\frac{M}{EI}$ - diagram due to the cantilever uplift of the trial reaction acting alone, and locate its center of gravity, distant from the right end, as shown below the horizontal line in Fig. 5(b). The Trial Δ_R and its uplift at the end of the beam ($= b \times$ Trial Δ_R) can now be added to the traverse as shown in Fig. 5(c).

The true value of R necessary to raise the right end of the beam to its settled position is now given by the proportion, $\frac{\text{True } R}{\text{Trial } R} = \frac{a\,\Delta_p - 1 \text{ in.}}{b \times \text{Trial } R}$. With the value of R determined, the moments may be computed by statics.

Traverse Method a Substitute for Theorem of Three Moments

Fig. 6 represents a series of continuous equal spans with a moment at the right end. The traverse method furnishes a quick and easy solution for all the moments and all the slopes of the elastic curve over the supports by the following construction: Beginning at the left end of Fig. 6(*c*) draw

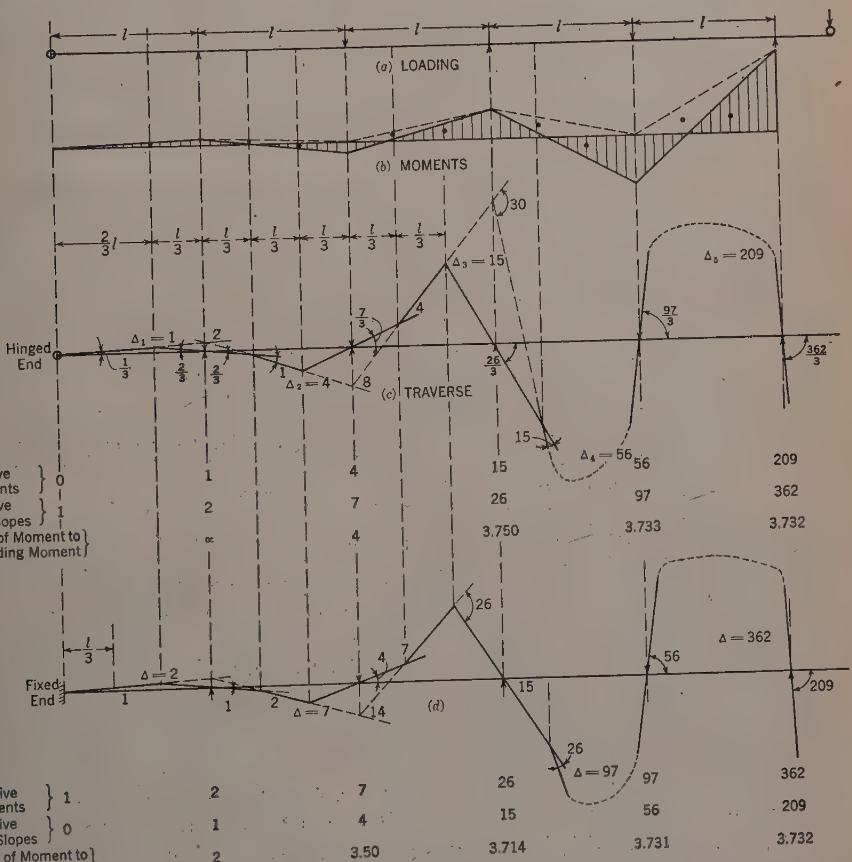

FIG. 6.

a traverse triangle for the flexure of the elastic curve of the left span. A low altitude for this triangle should be used, but no particular scale for the angles is necessary. Produce the long leg of the diagram to Point 2 over the second support and produce the short leg to the point marked 1 at the one-third span length of the second span. Draw Line 2-1-4-8. Through

Point 4 draw Line 4-4 passing through the third support. Draw Line 8-4-15, etc. The result is a series of "pennant" diagrams. Each angle marked Δ is a measure of the moment at the support to its right. The position of Δ above or below the horizontal through the supports shows whether the moment is positive or negative. All angles in Fig. 6(*c*) can be evaluated quickly by simple summation and by determining the products of angles and horizontal distances. For example, to find Δ_2,

$$\frac{2}{3}(l) + 1\left(\frac{2}{3}l\right) - \Delta_2\left(\frac{1}{3}l\right) = 0$$

Therefore, $\Delta_2 = 4$.

Each line through a support is tangent to the elastic curve. Its angle with the horizontal is the measure of the slope of the elastic curve at the support and is obtained by summation. For example, the relative slope over the third support is, $\frac{2}{3} + 1 - 4 = -\frac{7}{3}$. In Fig. 6(*c*) the value of each angle is shown at the angle point. Signs are omitted as unnecessary because the picture of the flexure discloses the direction of moments. Below Fig. 6(*c*) are shown the relative magnitude of the moments at the supports, the relative magnitude of the end slopes, and the ratio of each moment to the preceding moment. Fig. 6(*d*) shows the traverse for the same system with the left end fixed. Note that the numerical series for moments in Fig. 6(*c*) is the same as that for end slopes in Fig. 6(*d*), and *vice versa*. The moment at any support is now known in terms of the moment at the right end. For example, in Fig. 6(*c*), if the right end moment is 3 000, the moment at the second support will be $\frac{1}{209} \times 3\,000$.

If the number of spans is infinite the ratio of each moment to the preceding moment will be 3.73205. The difference between this ratio and the ratios at the fourth support in Fig. 6 is negligible in practical design.

The application of the traverse method to unequal span lengths and loaded spans can be demonstrated by a problem solved previously[4] by L. H. Nishkian and D. B. Steinman, Members, Am. Soc. C. E. For the present purpose, Fig. 7 need not be, and is not, drawn to scale. Free-hand sketching is recommended in practical work. It is known that negative moments will occur over the supports. Sketch in the four moment triangles, Δ_1, Δ_2, Δ_3, and Δ_4, illustrating the existence of such moments. Draw simple moment diagrams for the loading of each span using either the upper lines of the negative moment triangles for bases, as shown, or, if desired, using the horizontal line through the supports as a base. It will make no difference in the computations which is used.

Draw verticals through the centers of gravity of the negative moment triangles (at the third points of the beam) and through the centers of gravity

[4] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 10, Fig. 11.

of the simple moment diagrams, which, in this case, are at the centers of the beam. Draw the T-lines (transposition lines[5]) as shown.

To construct the traverse, Fig. 7(c), draw Line R_1 A_1, making a small angle with the unsprung beam. Draw Line A_1 Δ_1 which will pass over the second support and extend it to the intersection with the T-line at $\Delta_1 + \Delta_2$.

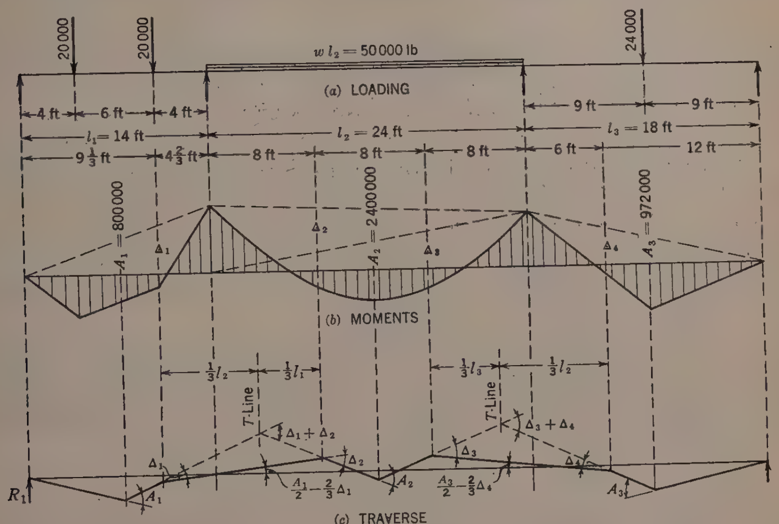

FIG. 7.

Draw Line Δ_1 Δ_2 passing through the support and extended to the vertical through the one-third length of the second span. Draw the line from $\Delta_1 + \Delta_2$ through Δ_2 to the vertical through the center of the span at A_2. Draw Line A_2 Δ_3 to pass over the third support, intersecting the T-line at $\Delta_3 + \Delta_4$. Continue this line sequence to the right end of the beam. Fig. 7(c) can now be solved for the values of the Δ-angles by giving the A-angles values that equal the respective areas of the simple moment diagrams and observing that $\frac{\Delta_1}{\Delta_2} = \frac{l_1}{l_2}$ and $\frac{\Delta_4}{\Delta_3} = \frac{l_3}{l_2}$. Solving for these Δ-angles and reducing them to their corresponding moments, the moment over the second support is found to be 97 166 ft-lb and the moment over the third support, 92 666 ft-lb. The T-lines are on the centers of gravity of the combined abutting negative moment triangles and this is what accounts for their mechanical significance.

The traverse method can be used for spans of variable moment of inertia by the use of trial end moments to locate the Δ-points and to obtain the relative $\frac{M}{EI}$- areas for equal end moments. Tables introduced[6] by Walter Ruppel, Assoc. M. Am. Soc. C. E., will facilitate this work for the cases

[5] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 3.

[6] *Loc. cit.*, pp. 167–187.

to which the tables apply. Tables that give $\frac{M}{EI}$ - areas for simple moments are unknown to the writer. Settlement of supports and side sway can be incorporated in the geometry of a traverse.

For the design of a member in a building of many stories and many bays, traverses in the form of Fig. 6 can be distributed vertically and horizontally to the columns and girders at the ends of the member. If the columns are flexible as compared with the girders they may be assumed to be fixed at floors above and below the joint in question, while the girders may be assumed to have a ratio of moment at the joint in question to moment at the joint one panel length distant, based on the factors indicated by Fig. 6 modified by an allowance for the joint stiffening effect of the columns. It will be found that a member can be designed quite satisfactorily by using moderately accurate judgment as to these moment ratios. If precision is desired and time is available the traverses may be extended two panel lengths, or more, from the member under consideration.

For the derivation of the basic formula for moments in single-span beams, fixed at one or at both ends, the traverse method offers alternate solutions which require a small fraction of the effort involved in the method of successive integrations found in textbooks on mechanics.

Conclusions

The analysis of continuous structures by traversing their elastic curves offers solutions that are easily understood and for which diagrams that clearly illustrate the problems can be easily drawn. The key constants for the analysis are simple $\frac{M}{EI}$ - and cantilever $\frac{M}{EI}$ - diagrams the construction of which is taught in the first lessons on beam stresses.

The method relieves the analyst from remembering or holding for reference the various forms of the three-moment equations or the slope-deflection equations. It requires no more analytical effort or labor of computation than these or other previously used methods.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

RELATION BETWEEN RAINFALL AND RUN-OFF FROM SMALL URBAN AREAS

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F. L. Flynt,[2] Assoc. M. Am. Soc. C. E.

Synopsis

The results of research into the relation between rainfall and run-off from small urban areas in St. Louis, Mo., are here presented as specific studies of the run-off from parts of two different city blocks tributary to street inlets and from both roofs and ground surface of another entire city block. The information submitted results from measurements of rainfall and storm flow for practically all heavy rains occurring from 1914 to 1933. The ratio of run-off to rainfall, defined in several ways, is shown to vary over a wide range.

Rainfall rates at each of the locations studied are reduced and developed into frequency diagrams, and these three rainfall studies, with one other, are combined into a master frequency study for the general region. Run-off is also studied as an independent phenomenon; the run-off frequency curves are developed in a form similar to the rainfall diagrams.

The two sets of curves are considered to be comparable as representing equivalent probabilities of occurrence. Ratios are then developed between corresponding values. Within certain limits, it is suggested that these ratios may be applied to proper rainfall frequency curves for other localities and will give approximate run-off values for similar conditions of surface.

Suggestions are offered as to how the values determined for specific blocks in St. Louis might be modified further to be applicable to (*a*) different surface slopes; (*b*) other percentages of impervious area; and (*c*) other typical soils. For Class (*c*), adaptation factors are determined by sprinkling, at definite rates, a number of segregated areas of bare soil and of turf.

Note.—Discussion on this paper will be closed in January, 1935 *Proceedings*.

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Introduction

In 1908, one of the writers had developed an application of the rational method of sewer design, for the City of St. Louis. The form of treatment and the mechanics of the application were considered satisfactory; but there was little information then available as to necessary ratios between rainfall and run-off under specific conditions. A proper development of the hydrology of urban drainage required the provision of dependable factors indicating the relation between rainfall and run-off for a wide range of situations.

Funds were made available by the City of St. Louis for a research program. In 1910, three tipping-bucket rain gauges were installed in the 500-acre Clarendon drainage area, (see Points 1, 2, and 3, Fig. 1) and about

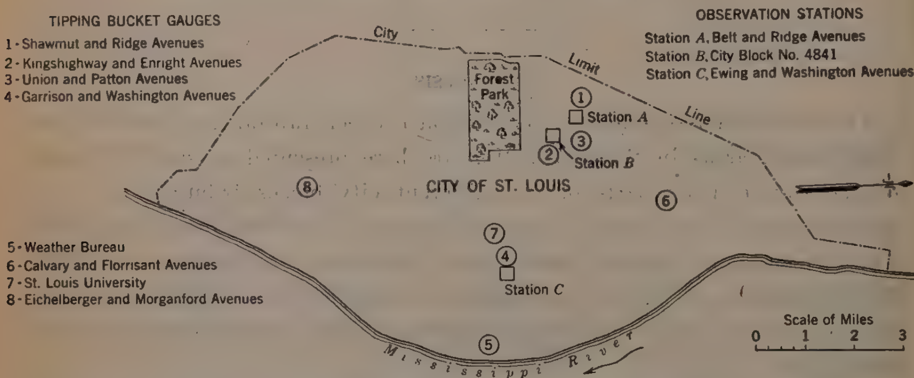


Fig. 1.—Locations of Observation Stations and Tipping-Bucket Rain Gauges.

ten water-level gauges in its main trunk sewer. The information thus secured was analyzed in part in 1915 and, again, in 1920, but not exhaustively. Tentative values were deduced and the resulting run-off factors have been used in St. Louis, but, heretofore, no thorough study has been completed to a degree justifying publication of the results.

When the characteristics of the flow in the main trunk sewer came to be scrutinized, it was apparent that a closer view of the basic relations might be secured from an examination of the run-off phenomena from smaller areas. Therefore, a study was undertaken of two inlet areas at Belt and Ridge Avenues and at City Block No. 4841 (Stations *A* and *B*, Fig. 1), involving the run-off from entire city blocks, exclusive of that from the roofs of residential buildings. Later, a gauge was installed in a lateral sewer near Ewing and Washington Avenues (Station *C*, Fig. 1), and the entire run-off of a third block, including that from roofs, was made available for analysis. The result of observations in these three city blocks, during heavy rains, is presented herein.

In the course of this more detailed study, a better acquaintance with the effect of soil characteristics and conditions upon run-off became desirable. Impressed with the simplicity of the sprinkling tests, made by the Miami

Conservancy District, a similar series of tests were conducted in St. Louis, using somewhat more elaborate and refined methods. The result of a part of this work has been published.[3] All the data are given in the unpublished Appendices filed in Engineering Societies Library, 29 West 39th Street, New York, N. Y. Included with these records is also the result of a similar series of tests, conducted under the supervision of one of the writers, in connection with the studies for storm-sewer design for Dallas, Tex.

PART 1.—Study of the Run-Off Factor by the Unit-Graph Method

Instruments and Installations.—For each of the three city blocks (Stations *A*, *B*, and *C*, Fig. 1). the pertinent information as to surface condition, slopes, etc., is shown in Figs. 2 to 5, inclusive, and in Table 1. During

TABLE 1.—Basic Information Pertaining to Areas *A*, *B*, and *C*, Fig. 1

Type of area	Total Area Tributary to the Gauge: As of 1917: Subdivided, in square feet	As of 1917: Total in: Square feet	As of 1917: Total in: Acres	As of 1917: Total in: Percentage	As of 1933: Subdivided, in square feet	As of 1933: Total in: Square feet	As of 1933: Total in: Acres	As of 1933: Total in: Percentage	Area in Entire Block: Subdivided, in square feet	Area in Entire Block: Total in: Square feet	Area in Entire Block: Total in: Percentage
(*a*) Area *A* (Belt and Ridge Avenues; See Fig. 2)											
Impervious:											
Streets and sidewalks	16 117										
Alleys	6 098										
Roofs and porches	4 356	42 255	0.97	42	......	....	1.07	49	.....	....	.....
Sheds	3 485										
Paved yards and walks	12 199										
Pervious	......	57 935	1.33	58	......	....	1.13	51	......	....	.....
Total	......	....	2.30	100	......	....	2.20	....	......	....	.....
(*b*) Area *B* (City Block No. 4841; See Fig. 3)											
Impervious:											
Streets and sidewalks	4 872								64 030		
Alleys	14 400								14 400		
Roofs and porches	1 932	40 916	....	29	......	46 021	....	32	81 126	199 737	51.8
Sheds	7 673								8 691		
Paved yards and walks	12 039								31 490		
Pervious	......	101 442	....	71	......	96 137	....	68	......	186 341	48.2
Total	......	142 358	3.27	100	......	142 158	3 27	100	......	386 078	100.0
(*c*) Area *C* (Ewing and Washington Avenues; See Fig. 4)											
Impervious:											
Buildings	58 588				59 370						
Sheds	7 072				6 618						
Walks	39 888	136 576	3.14	72.4	39 888	136 904	3.14	72.1	......	....	.....
Alleys	7 725				7 725						
Streets	23 303				23 303						
Pervious	......	52 064	1.20	27.6	......	52 846	1.22	27 9	......	....	.....
Total	......	188 640	4.34	100.0	......	189 750	4.36	100.0	......	....	.....

the period of the tests, a few changes took place in the buildings and garages, the only one of importance resulting from the erection of an apartment house at the corner of Belt and Ridge Avenues. The data for the beginning and the ending of the test period are shown in Table 1.

[3] *Municipal and County Engineering*, December, 1922.

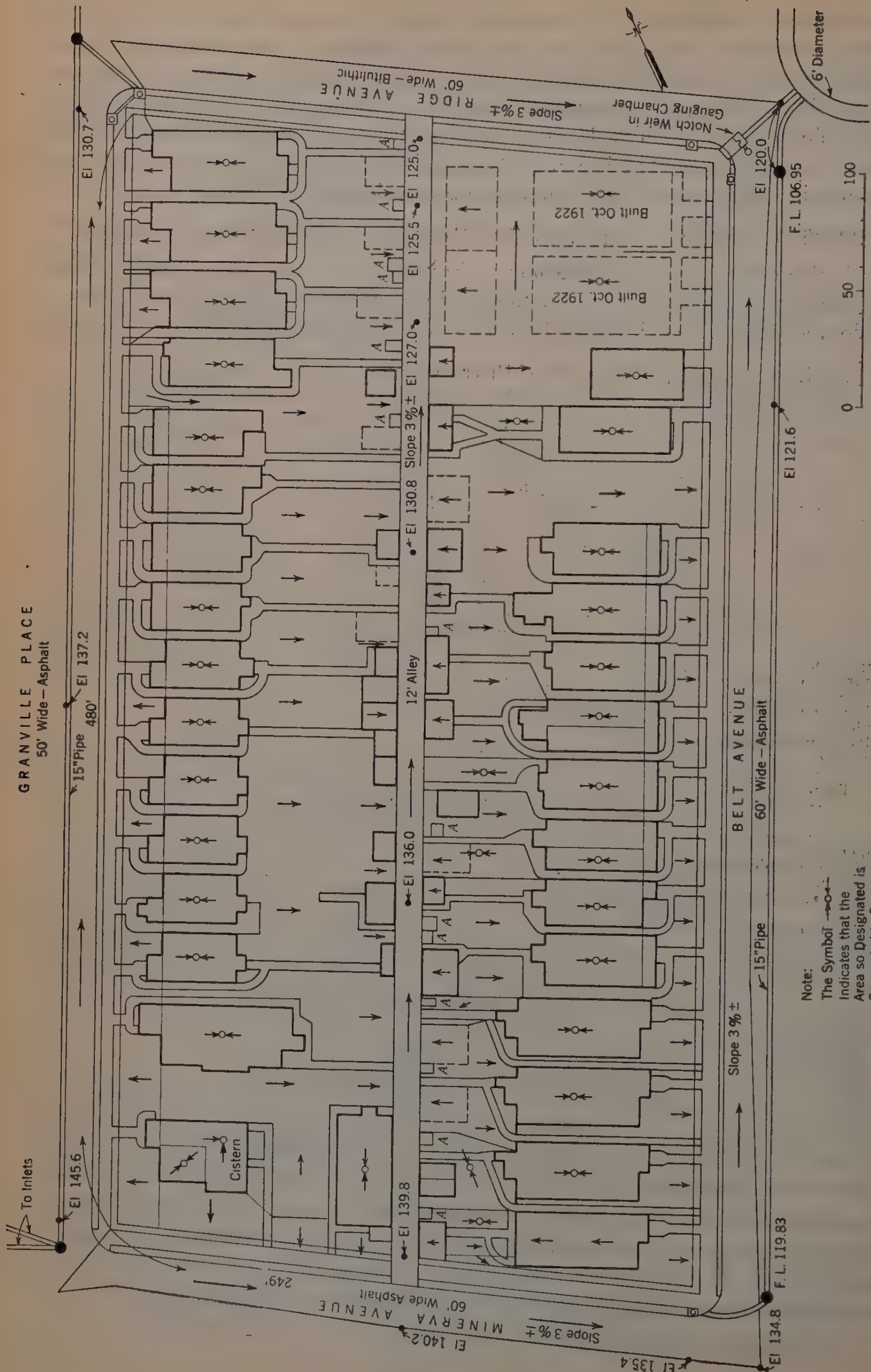


FIG. 2.—AREA TRIBUTARY TO STATION A AT BELT AND RIDGE AVENUES (SEE TABLE 1 (*a*)).

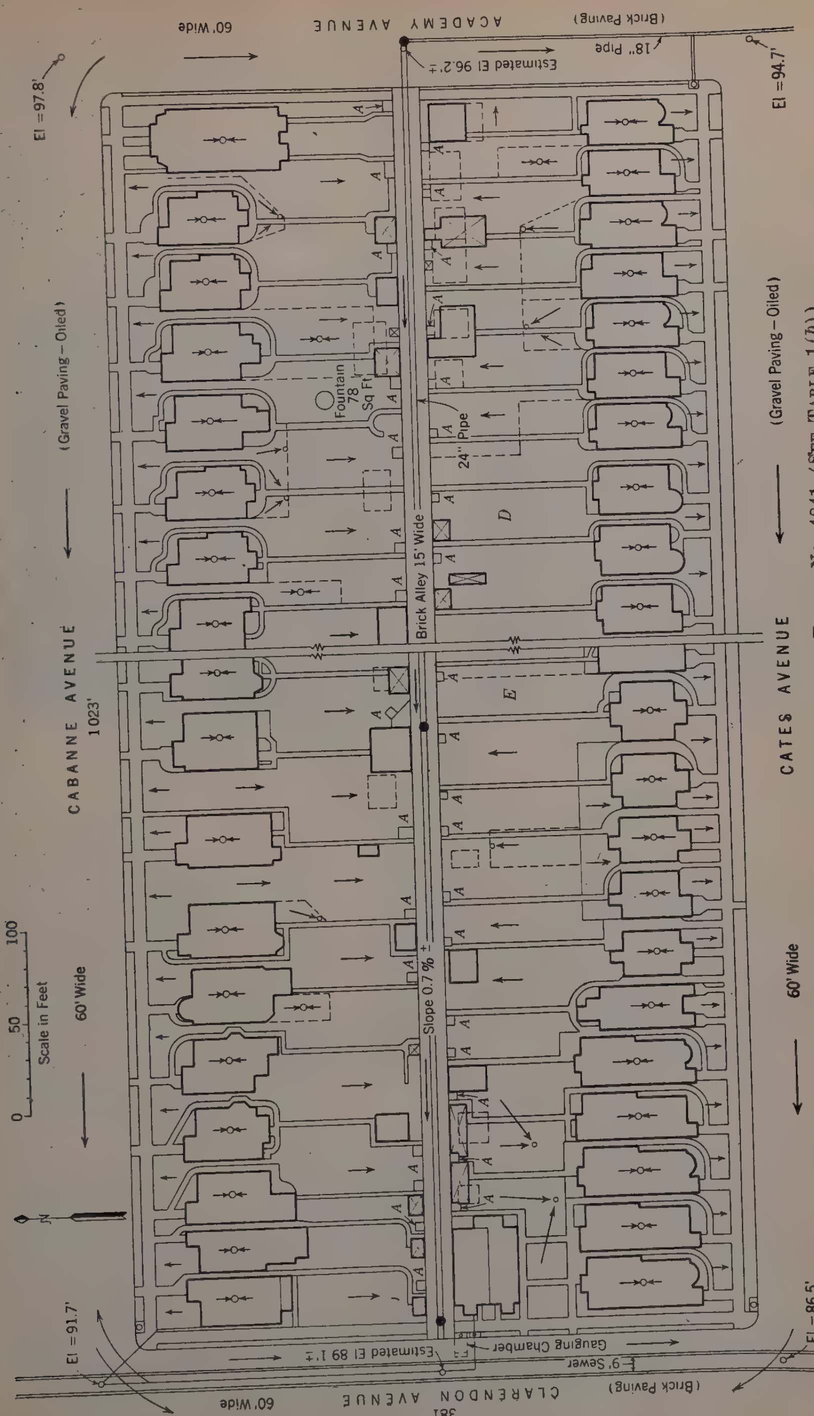


FIG. 3.—AREA TRIBUTARY TO STATION B AT CITY BLOCK No. 4841 (SEE TABLE 1(b)).

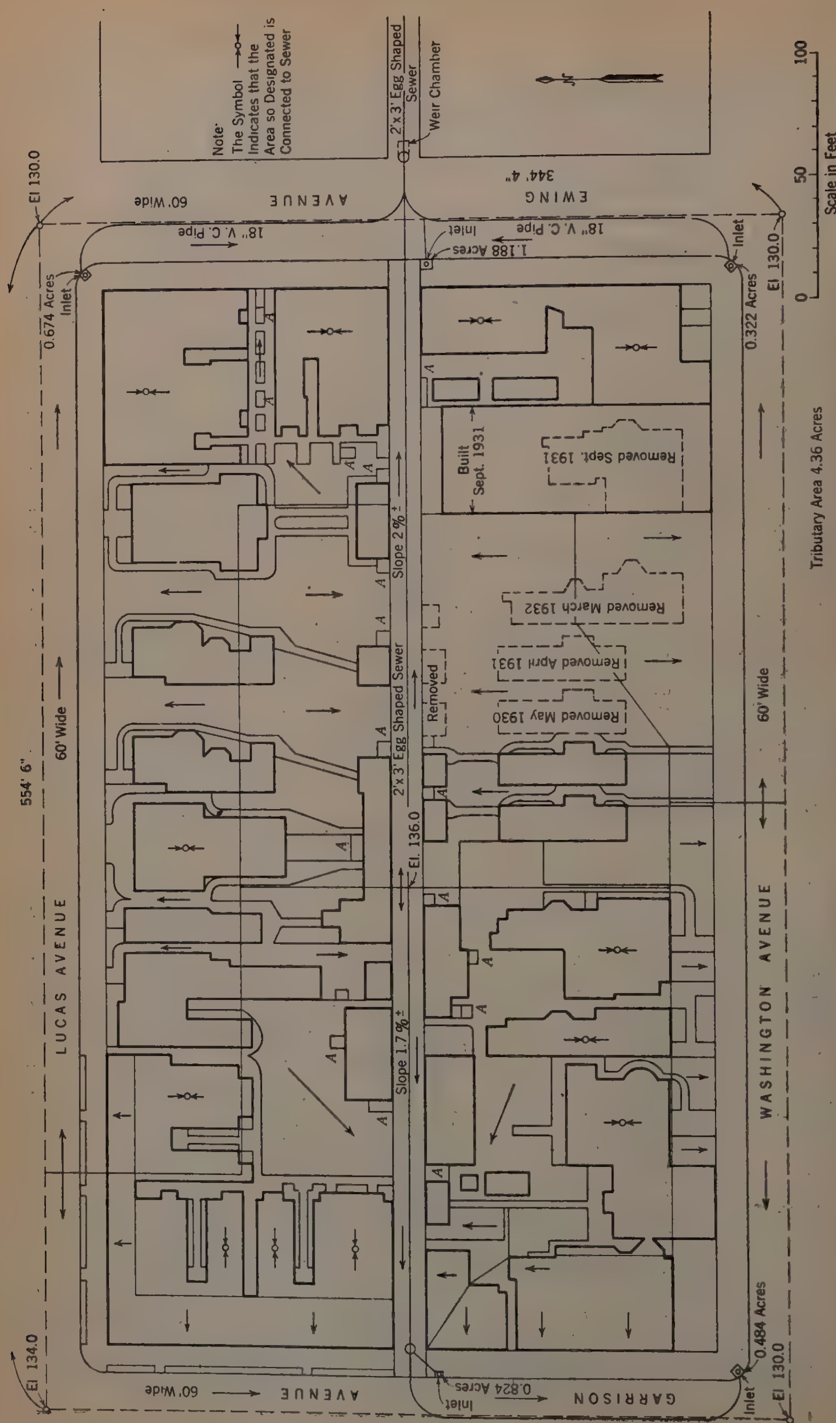


FIG. 4.—AREA TRIBUTARY TO STATION *C* AT EWING AND WASHINGTON AVENUES (SEE TABLE 1(*c*)).

Description of Instruments for Inlet Area Tests.—At each location the run-off was measured by recording the head on a 90°, V-notch weir of $\frac{3}{8}$-in. steel plate with bronze edges. At Station *A* and for Station *B*, the weir was installed in a special chamber built as an extension of the bowl

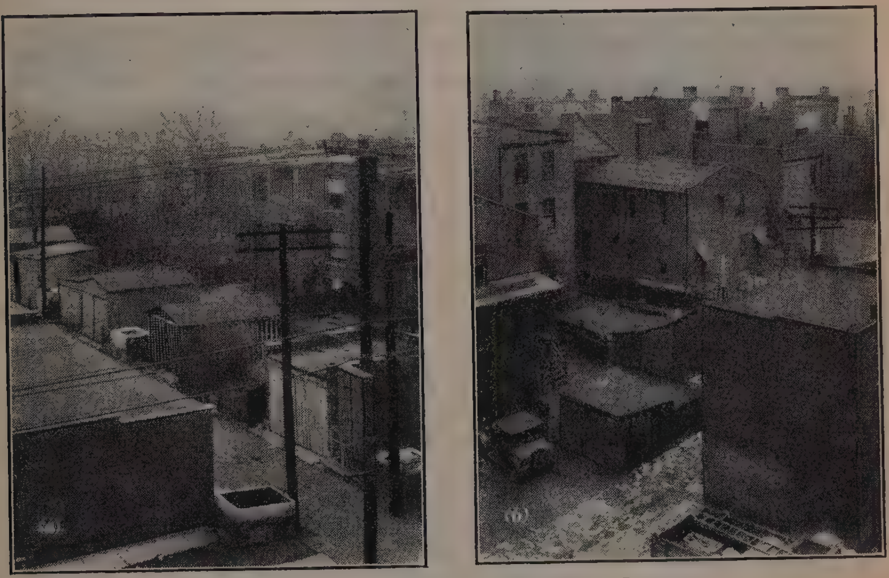


FIG. 5.—TYPICAL APPEARANCE OF PROPERTY INVOLVED: (*a*) AREA *A*; (*b*) AREA *C*.

of the catch-basin. At Station *C*, the weir was installed in a chamber built into the brick sewer itself. In each instance, the head on the weir was measured by a pressure bulb connected with a recording clock gauge.

In Fig. 2, 30% of the area above the inlet at the northwest corner of Belt and Minerva Avenues is assumed to be tributary to the gauge at Belt and Ridge Avenues for all rains. Referring to Fig. 3, the area marked, *D*, was judged to yield practically no run-off. The yard marked, *E* (Fig. 3), was found to have several small sumps, so that water from this yard did not reach the gauge. Consequently, it was assumed that water from all storms would probably stand.

At Station *C* (see Fig. 4), the rainfall was measured by a standard tipping-bucket gauge on a building adjacent to the upper edge of the block. At Stations *A* and *B*, home-made rain gauges were installed in such a way that the rain water falling into the gauge was measured as head on an additional pressure bulb, which actuated an extra pen installed on the clock gauge. These rain gauges were calibrated by introducing known quantities of water into the stand-pipe and noting the chart records. To supplement these gauges, three tipping-bucket rain gauges were also available at Shawmut and Ridge Avenues, at Kingshighway and Enright, and at Union and Patton Avenues, all within a mile of the two home-made gauges.

Pervious Area

At Area *A*, Fig. 2, the pervious part consists of small front lawns, some good turf, and some bare soil. At Area *B*, Fig. 3, nearly all the pervious part is well turfed. At Area *C*, Fig. 4, the pervious part consists generally of some small separated plots of rather hard-packed soil.

Reduction of Records.—An examination of a typical chart for Belt and Ridge Avenues (Fig. 6) shows the rain pen moving from the outer circle

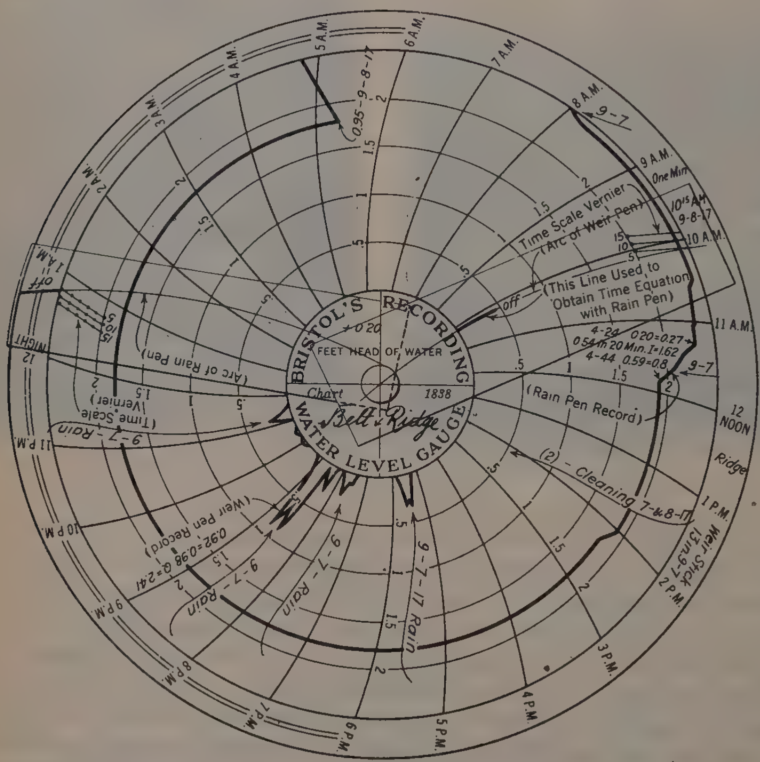


Fig. 6.—Rainfall and Run-Off Record, Station *A*; Rain of September 7, 1917.

toward the center, and the weir pen moving outward from the center. Celluloid verniers, indicated by the shaded rectangular areas, make it possible to read the time scales to the nearest minute. Since the rain pen is a home-made addition to the gauge, it does not follow the printed time lines, and the apparent time of the rain record must be corrected to clock time. Fig. 7 shows the curves obtained by plotting the information obtained from charts, such as Fig. 6. The records for City Block No. 4841 were similarly reduced and plotted; the tipping-bucket rain gauge at Station *C* necessitated some differences in the detail of handling its records.

Run-Off Characteristics.—To introduce the factors that enter into the rainfall-run-off problem, the following discussion of a hypothetical case is offered.

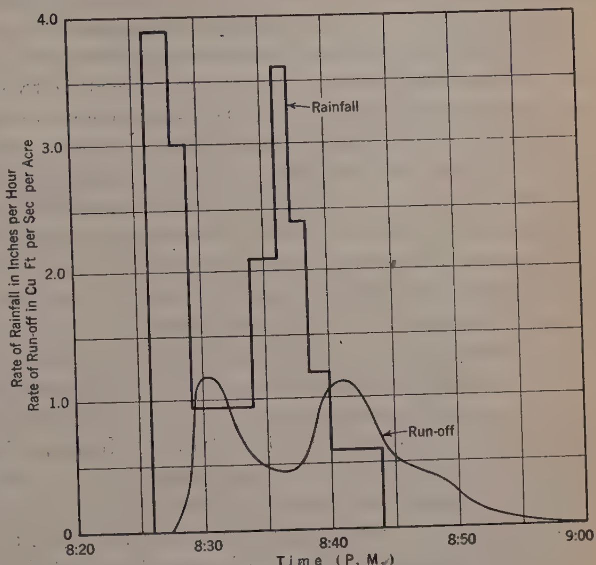


FIG. 7.—RAINFALL-RUN-OFF RATE CURVES; RAIN OF SEPTEMBER 7, 1917, AT STATION A.

Fig. 8 represents the case of an inclined plane with a pervious surface, exposed to a rainfall of uniform rate. Let i = intensity, or rate of rainfall, in inches per hour; a = rate of absorption, in inches per hour; e = rate of evaporation in inches per hour; I = residual rainfall available for run-off = $i - (a + e)$; f = depth of water film on the surface due to rainfall; f_c = critical depth of film when run-off begins; $V \doteq$ velocity of water in the

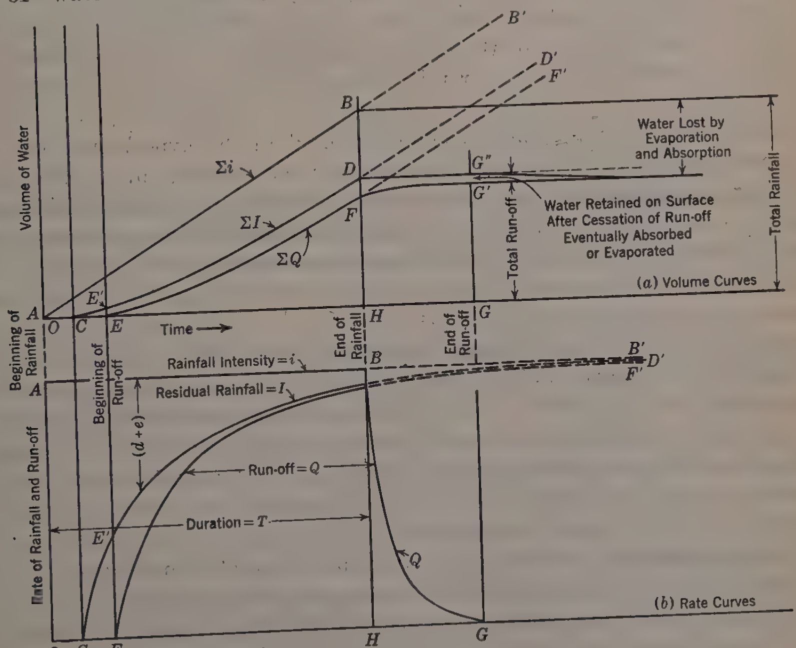


FIG. 8.—RELATION BETWEEN RAINFALL AND RUN-OFF.

moving film; Q = rate of run-off; and t = duration of a rain. Point A represents the beginning of the rainfall. For a time, the rate, $(a + e)$, is greater than i and no water can accumulate on the surface; but since i is constant and $(a + e)$ is decreasing, eventually $(a + e) = i$, as at Point C, Fig. 8, when the surface film begins to form and I begins to have a positive value. The water in this film is subject to two opposing forces: (1) A component of the force of gravity which tends to cause motion on the plane; and (2) surface tension which tends to oppose this motion. As the rate, $(a + e)$, decreases, the depth of film, f, increases until the critical depth, f_c, is attained, as at Point E, Fig. 8, when run-off begins. Under the assumed conditions, run-off would begin simultaneously over the entire area.

Frictional resistance, which tends to reduce acceleration, increases with the velocity and decreases with the depth. Therefore, the values of f and V at any given point on the plane depend on a balance between these opposite effects combined with the effects of absorption and evaporation.

If the assumed conditions continued for an indefinite time, the value of $(a + e)$ would approach zero as a limit, and the values of I and Q would approach i as a limit, as indicated in the diagrams by Points B', D', and F'. Fig. 8. From this time on, the product, $f \times V$, at all points on the plane would be constant, as would the total volume of water in transit, represented by the intercept on the ordinates between the lines, CD and EF, on Fig. 8(a).

If, however, rainfall should cease after a time, t, the value of i would drop immediately to zero as indicated by the line, BH, in Fig. 8(b). The water in transit, represented by the line, DF, in Fig. 8(a), would still be available for run-off which would continue at a decreasing rate until the moving film was reduced to the critical depth, f_c, as at Point G, when run-off would cease; the water retained would eventually be absorbed or evaporated.

Actual Areas.—Many factors not considered in the hypothetical case are involved in an actual inlet area. It is not a plane surface, but a collection of small contiguous planes; the slopes may range from horizontal for tennis courts, to vertical for building walls; the permeability of the component surfaces may likewise vary over a wide range, from asphalt street paving to grass plots and cultivated ground. The actual area may contain well-defined watercourses, such as gutters and rivulets.

Depressed areas may be present into which water drains and thus becomes trapped; this drainage never reaches the inlet, but eventually is absorbed or evaporated. Such a condition may be designated as retention by pondage. If the depression is shallow, it may soon be filled and then contribute its overflow to the run-off; or, the depression may have an outlet that is too small to drain the water as fast as it falls. This water will appear in the total run-off, but its detention will have an effort upon the rate of run-off.

Although an actual inlet area may be composed of elemental plane surfaces, it is impracticable to analyze run-off from each small area separately. The entire inlet area is here taken as a unit, and the run-off from it is studied as a whole.

Unit Graph.—Fig. 9 is a typical example of a rainfall-run-off graph as derived from the local records. A study of these graphs shows that the reduction in height of the run-off peaks, as compared with the rainfall peaks, is not entirely due to the loss of water by absorption and evaporation. It is

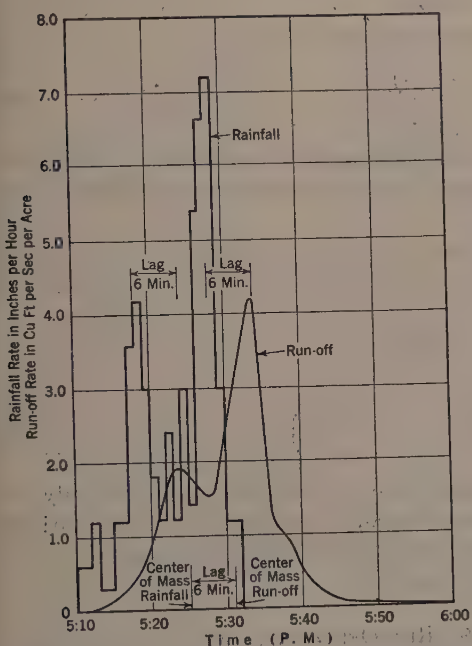


FIG. 9.—RATE CURVES FOR RAINFALL AND RUN-OFF, STATION C; RAIN OF SEPTEMBER 7, 1920.

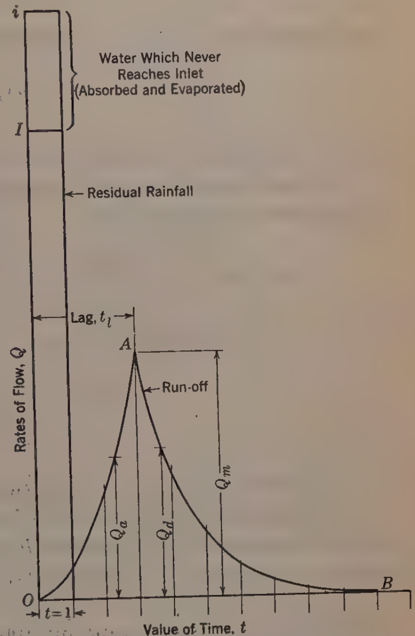


FIG. 10.—UNIT GRAPH REPRESENTING THEORETICAL RUN-OFF RESULTING FROM RAINFALL ON UNIT AREA FOR UNIT TIME.

due partly to the fact that run-off is spread over a greater time interval than rainfall. If some method could be found to separate these two rate-reducing effects of loss by absorption and of distribution of the run-off in time, it would be possible to measure the absorption and evaporation characteristics of the areas under investigation.

Following is an outline of a method of analysis based upon a theoretical unit graph showing the probable distribution in time of the run-off resulting from 1 min of uniform rainfall.

In Fig. 10, the rectangle, Oi, represents the volume of 1 min of uniform rainfall. The rectangle, OI, represents the volume of this rainfall which will ultimately reach the sewer.

The shape of the run-off unit graph was derived from a study of the records of a few short rains of fairly uniform intensities, the equations for the two branches being entirely empirical. The quantities, Q_a and Q_d, respectively, are the ascending and descending instantaneous values of Q resulting from a rainfall of unit duration on a unit area; Q_m is the maximum value of

Q_a and Q_d; t is the time measured from the beginning of the rainfall; t_l is the lag, or the value of t at Q_m; and j and k are arbitrary constants. For the increasing values of Q, the equation of the line, OA, is:

$$Q_a = Q_m \left(\frac{t}{t_l}\right)^j \qquad (1)$$

For decreasing values, the equation of the line, AB, is:

$$Q_d = \frac{Q_m}{k^{t-t_l}} \qquad (2)$$

From Equations (1) and (2) it is seen that when $t = t_l$, $Q_a = Q_d = Q_m$.

Integrating and adding Equations (1) and (2), the area under the unit run-off curve is found to be:

$$I = Q_m \int_0^{t_l} \left(\frac{t}{t_l}\right)^j dt + Q_m \int_{t_l}^{\infty} \frac{dt}{k^{(t-t_l)}} = Q_m \left(\frac{t_l}{j+1} + \frac{1}{\log_e k}\right) \qquad (3)$$

Since $I = Pi$, the value of Q_m is found to be:

$$Q_m = \frac{Pi}{\left(\dfrac{t_l}{j+1} + \dfrac{1}{\log_e k}\right)} \qquad (4)$$

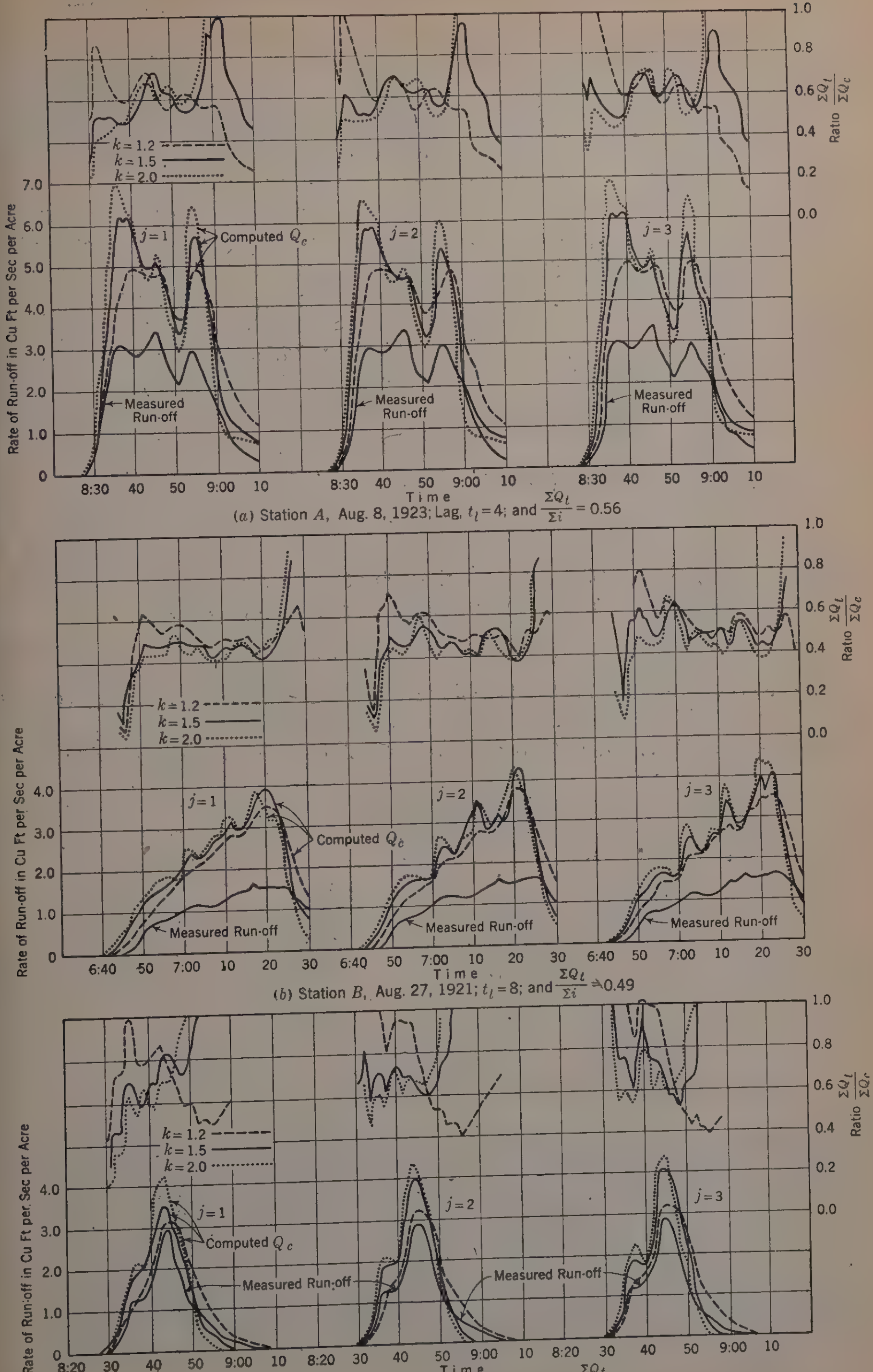
The values, j and k, in Equations (1) to (4) can be determined only by trial. After many rains had been investigated, it was found that, for Stations A and B, values of $j = 1.0$ and $k = 1.2$ and, for Station C, values of $j = 2$ and $k = 2$ gave the best results (see Fig. 11).

TABLE 2.—COMPUTATION OF 100% RUN-OFF CURVE, OR VALUES OF Q_c (SEE FIG. 12)

(ASSUME $t_l = 3$; $j = 2$; AND $k = 2$)

Time, t, in minutes of run-off	Quantities of run-off, Q, at successive values of time, t, in minutes					100% run-off, Q_c	Time, t, in minutes of run-off	Quantities of run-off, Q, at successive values of duration period, I, in minutes					100% run-off, Q_c
	1	2	3	4	5			1	2	3	4	5	
	and corresponding values of rainfall intensity, i, in inches per hour							and corresponding values of rainfall intensity, i, in inches per hour					
	2	4	6	3	1			2	4	6	3	1	
1......	0.092	.....	.....	.....	.....	0.092	10.....	0.007	0.026	0.077	0.077	0.052	0.239
2......	0.364	0.184	.....	.....	.....	0.548	11.....	0.004	0.013	0.039	0.039	0.026	0.121
3......	0.820	0.728	0.276	.....	.....	1.824	12.....	0.002	0.007	0.020	0.020	0.013	0.062
4......	0.410	1.640	1.092	0.138	.....	3.280	13.....	0.001	0.004	0.010	0.010	0.007	0.032
5......	0.205	0.820	2.460	0.546	0.046	4.077	14.....	.....	0.002	0.005	0.005	0.004	0.016
6......	0.103	0.410	1.230	1.230	0.182	3.155	15.....	.....	0.001	0.003	0.003	0.002	0.009
7......	0.052	0.205	0.615	0.615	0.410	1.897	16.....	.....	.....	0.002	0.002	0.001	0.005
8......	0.026	0.103	0.308	0.308	0.205	0.950	17.....	.....	.....	0.001	0.001	.....	0.002
9......	0.013	0.052	0.154	0.154	0.103	0.476	18.....	.....	.....	.....	.....	.....	.....

Fig. 12 and Table 2 show the method of computing the ordinates and plotting the 100% run-off or Q_c-curve for a short hypothetical rain. The same method may be applied to any rainfall record to find the computed 100%



(a) Station A, Aug. 8, 1923; Lag, $t_l = 4$; and $\frac{\Sigma Q_t}{\Sigma i} = 0.56$

(b) Station B, Aug. 27, 1921; $t_l = 8$; and $\frac{\Sigma Q_t}{\Sigma i} = 0.49$

(c) Station C, Sept. 7, 1917; Lag, $t_l = 6$; and $\frac{\Sigma Q_t}{\Sigma i} = 0.74$

FIG. 11.—VARIATIONS OF COMPUTED 100% RUN-OFF FOR DIFFERENT VALUES OF j AND k.

run-off for any area for which the lag is known. The computed Q_c-curve shows the rate-reducing effect of the distribution of run-off, in time, separated from the effect due to absorption and evaporation. A comparison of the measured run-off curve with the 100% run-off curve shows the effect of absorption and evaporation.

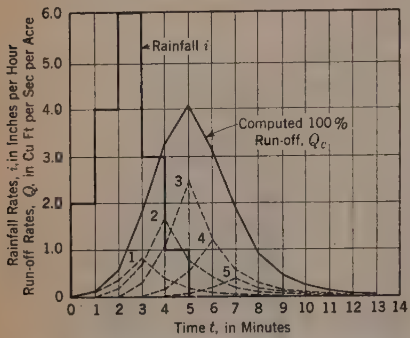


FIG. 12.—HYPOTHETICAL RAIN, SHOWING APPLICATION OF UNIT GRAPH (SEE TABLE 2).

Lag.—The term, "lag", as used herein has reference only to the difference in phase between salient features of the rainfall and run-off rate curves; its numerical values (which are generally somewhat less than the "time of concentration") are difficult to determine with the desired accuracy owing to the limitations of the recording instruments. (A full discussion of this subject is included in the record manuscript filed in Engineering Societies Library.) Observed values, subject to correction, are taken from the time difference between salient features of the rainfall and run-off rate curves; or between the centers of mass of the rainfall and run-off rate curves.

The comparatively wide range in the lag at each location led to the inference that the lag was a variable, its value being determined more by rainfall characteristics than by the characteristics of the drainage area. Diagrams were made of the relation between the lag and various rainfall characteristics (see Fig. 13), but no evidence of correlation was found. On all

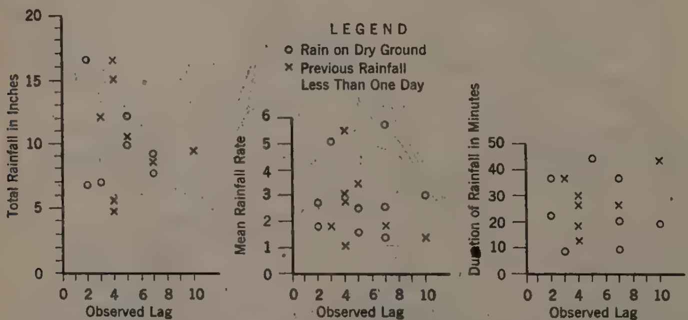


FIG. 13.—TYPICAL CORRELATION DIAGRAMS FOR LAG AT STATION *A*.

these diagrams, the values of the lag for each location tended to be concentrated about certain modal values. A statistical study (see Fig. 14) indicated that the mean, the median, and the modal values of the lag for each location were approximately equal, which suggested that the value was nearly constant for each station, and that the variations from the mean value were probably chance variations which were partly explained later by the discovery of small unsuspected time errors. Each of the plotted points through which the curves in Fig. 14 were drawn, represents the number of storms investigated which

had the corresponding observed value of lag, or less. Values of the lag as observed at the three observation stations are given in Table 3. Under certain circumstances, the true lag may vary slightly from the adopted values in Table 3, but the recording instruments available for this work are not sufficiently sensitive to measure the variation with any assurance of accuracy.

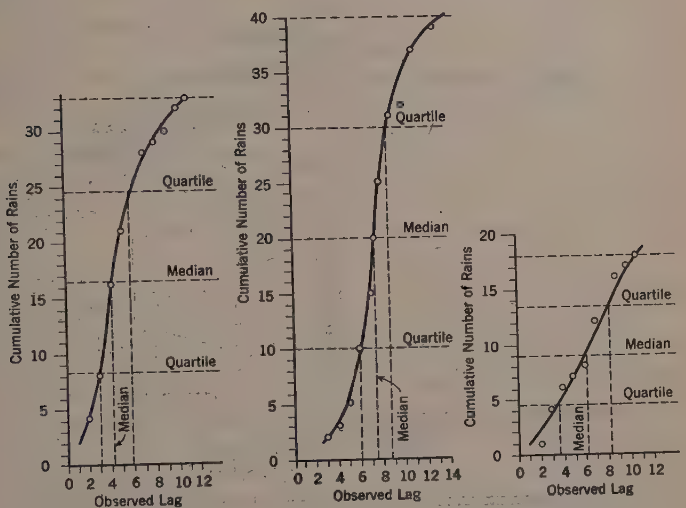


FIG. 14.—OGIVE CURVES SHOWING VARIATION IN OBSERVED VALUE OF LAG; (a) STATION A; (b) STATION B; (c) STATION C.

As only three different areas are represented in the local records, it is impossible to establish any definite rules for the determination of the lag for a given inlet area. The two stations for which the extreme values of the lag are indicated (Station A, 4 min; and Station B, 8 min) have widely different

TABLE 3.—VALUES OF LAG, t_l, IN MINUTES, OBSERVED AT THE THREE OBSERVATION STATIONS (SEE FIG. 1)

Station	Mean	Median	Mode	Adopted value
A	5.3	4.1	4.0	4
B	8.3	7.5	8.0	8
C	6.4	6.1	7.0	6

characteristics. The conditions of size and shape which tend to increase the lag are combined with conditions of slope which have the same tendency. This makes it difficult to separate their effects, but it does give an idea of the maximum range of value of the lag for similar areas.

Drainage Area C (Fig. 4) includes several independent inlet areas and considerable roof drainage, and the storm water from all parts of the block reaches the sewer after a comparatively short run. At the other stations, the flow is entirely over the surface to the point of measurement. For this

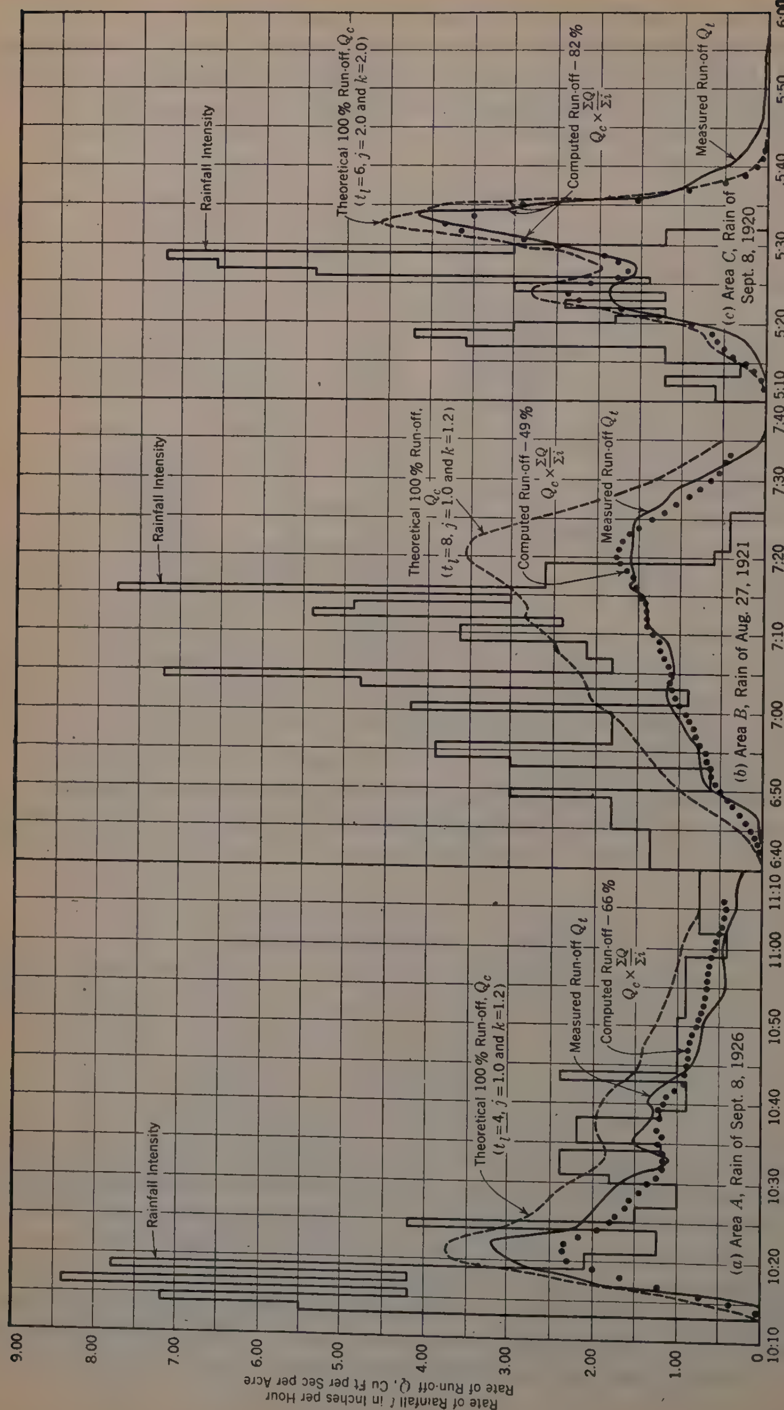


FIG. 15.—RATES OF RAINFALL, i, AND RUN-OFF, Q.

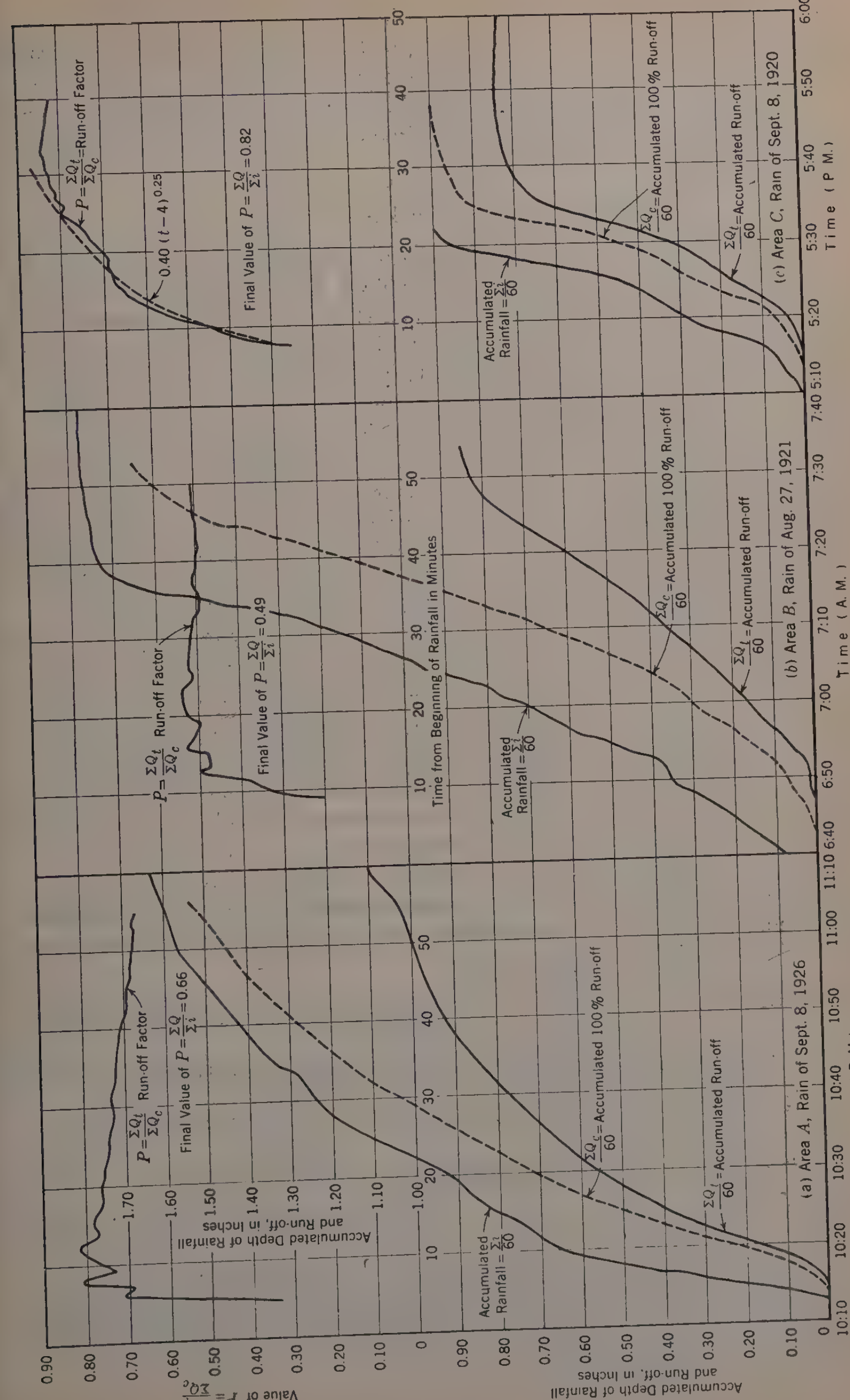


FIG. 16.—ACCUMULATED DEPTHS OF RAINFALL AND RUN-OFF AND VALUES OF P.

reason a direct comparison cannot be made with the other locations, but the results indicate that the lag for a completely sewered area is practically the same as that for a simple inlet area of comparable size.

The majority of inlet areas, at least in St. Louis, would have lags somewhere between 4 and 8 min. For any ordinary inlet area, a lag of 5 min could not be very far from the true value.

Determination of the Run-Off Factor

The 100% Run-Off Graph.—The rates of 100% run-off, computed by the same method as shown in Fig. 12 and Table 2, have been plotted as broken lines in Fig. 15 for a typical storm at each station. The lines represent the theoretical value of the run-off rate for an absolutely impervious area if there is no evaporation. Their close similarity to the measured run-off curves is obvious; the infrequent dissimilarities are due to several causes. For example, the factors, j and k, in the formulas may be slightly in error due to the difficulty of ascertaining these values. There is a possibility that the rate of rainfall for single minutes may be in error, although the average rate for several minutes may be correct. Furthermore, there may be slight time errors which would tend to shift the rainfall and run-off peaks one way or the other. All these causes would affect the shape of the Q_c or 100% run-off curve and, in some cases, their effects might be cumulative and appreciable.

Except as otherwise stated, the run-off factor, P, is defined as the ratio between ΣQ_t, the volume of water that has run off from the given area to a given time, and ΣQ_c, the volume of water that would have run off from the area during the same time had there been no evaporation or absorption. It is indicated most conveniently (see Fig. 16) by the ratio between corresponding ordinates of the mass curves of the measured run-off and the 100% run-off. (In Figs 16(a) and 16(b), the ΣQ_c-curve would eventually meet the Σi-curve if the entire record had been plotted.)

Typical Storms.—The storm of September 8, 1926, on Area A (see Fig. 15(a)) begins with a period of high intensity lasting for about 6 min; then it falls suddeuly to medium values which gradually decrease with some minor fluctuations until the end of the storm.

The ordinate to the mass curve, $\frac{\Sigma i}{60}$ (Fig. 16(a)) for any given time is proportional to the area of the rainfall-rate curve from the beginning of the rain to the given time. Similarly, the $\frac{\Sigma Q_c}{60}$ and $\frac{\Sigma Q_t}{60}$ -curves are the mass curves for the 100% run-off and the measured run-off, respectively.

The graph of the run-off factor, $P = \frac{\Sigma Q_t}{\Sigma Q_c}$, is obtained from the ratio of corresponding ordinates of the mass curves. The comparatively large difference between the maximum and final values of the run-off factor in this case is rather unusual and is due to the great and sudden change in rainfall intensity early in the storm.

An interesting type of rainfall (which seldom occurs more than once a year in St. Louis and then only between August 15 and September 15) is represented by the storm of August 27, 1921, on Area *B* (Figs. 15(*b*) and 16(*b*)). This type of storm, with its steadily increasing intensity, results in higher run-off rates than any other storm of comparable magnitude.

The run-off factor rises to a maximum value within about 20 min and remains practically constant for the remainder of the storm, indicating that once a certain degree of saturation has been reached, further increases in rainfall rates have little effect. The low value of *P* and the smoothness of the curve are characteristic of this area, which is large and flat.

The rain of September 8, 1920, on Area *C* (Fig. 15(*c*)), also begins with a low rainfall intensity and builds up to a climax. The high value of the run-off factor is characteristic of this location which is a completely sewered city block and not an inlet area as are the others.

Complete Data.—Curves similar to Fig. 15(*a*), prepared by the unit-graph method for all representative storms at each location, have been filed with the record manuscript in Engineering Societies Library. On these diagrams is shown, in step curves, the appropriate rainfall rate information; in dotted lines, the 100% run-off curve, Q_c, of the unit-graph method, using the values of j and k appropriate to the particular block under study; and the actual run-off rate curve as determined from the sewer gauges (Q_t). For ready reference, the end values are given on each chart; that is, $\frac{\Sigma Q_t}{\Sigma i}$, or total run-off divided by total rainfall. Notes of other prior precipitation that would have affected ground conditions are also given.

Unit-Graph Method of Determining Run-Off Factor.—These diagrams have been studied in a number of ways. The form of the rainfall-rate curve having been altered by the unit-graph application, it was expected that the ratio of simultaneous values of the measured run-off, Q_t, to the calculated run-off, Q_c, which could then be determined, would be found to have a fairly simple series of values and a definite relationship to certain remaining variables.

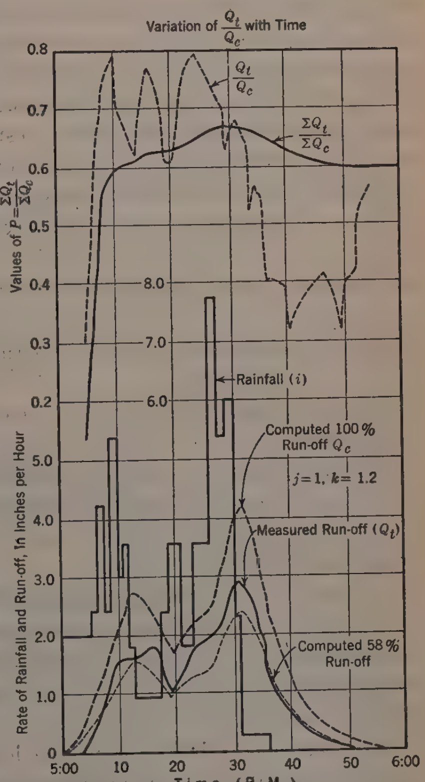


FIG. 17.—VARIATION OF Q_t/Q_c WITH TIME; AREA *A*, SEPTEMBER 8, 1920 (HEAVY RAIN 5 HOURS PREVIOUS; $t_i = 4$ AND $\frac{\Sigma Q_t}{\Sigma i} = 0.58$)

Ratio of Instantaneous Values.—In Fig. 17, the rate ratio, $\frac{Q_t}{Q_c}$, is compared with the volumetric ratio, $\frac{\Sigma Q_t}{\Sigma Q_c}$. The former is much more irregular than the latter, due to the causes enumerated under the heading, "Determination of the Run-Off Factor: The 100% Run-Off Graph". It will be noted in Fig. 17 that when the slope of the run-off curve is zero (at the peaks and valleys) as at 5.13 P.M., 5:19 P.M., and 5:31 P.M., the value of $\frac{Q_t}{Q_c}$ is nearly the same as that of $\frac{\Sigma Q_t}{\Sigma Q_c}$. A practical comparison can be made by ignoring exact time relations and comparing Q_t with Q_c only for generally corresponding peak values.

The results of such a study of all the data are plotted as percentile curves on Fig. 18(*c*). The values cover a wide range and no method has been found to co-ordinate them to other variables. In Fig. 18(*b*), the values are separated in accordance with time of occurrence after the beginning of the rain, but without distinctive results. It seemed possible that more definitely characteristic values for each block might result from the use of average peak values of 5-min duration rather than extreme peak values. The rates thus obtained are shown on Fig. 18(*d*) and may be compared to Fig. 18(*c*).

There is a marked correlation between the ratio of these 5-min peak rates and the final value of the run-off factor, $\frac{\Sigma Q_t}{\Sigma i}$. Correlation diagrams, of which Fig. 19 is an example, indicate that the final value of $P_1 = \frac{\Sigma Q_t}{\Sigma i}$, when applied to the 100% run-off curve, will give a computed peak run-off which should approximate the true value, except for peaks occurring early in the storm. This final value of P_1 can be determined easily and accurately by the ratio of the total run-off to the total rainfall. It is not affected by time errors or by uncertainty as to values of j and k in the unit-graph formulas, as are the other ratios studied.

If each of the ordinates of the 100% run-off curve (for a given storm and location), as computed by means of the unit-graph formula, is multiplied by the final value of the run-off factor for the given storm, the resulting curve usually agrees fairly well with the measured run-off curve, as shown by the circled points in Fig. 15.

The agreement is better in the longer storms, and for the later parts of storms, especially if the maximum rainfall intensity occurs after 15 or 20 min of rainfall when the run-off factor curve has flattened out, as in Fig. 15(*b*). When the maximum rainfall intensity occurs early in the storm, the agreement is not so good (see Fig. 15(*a*)).

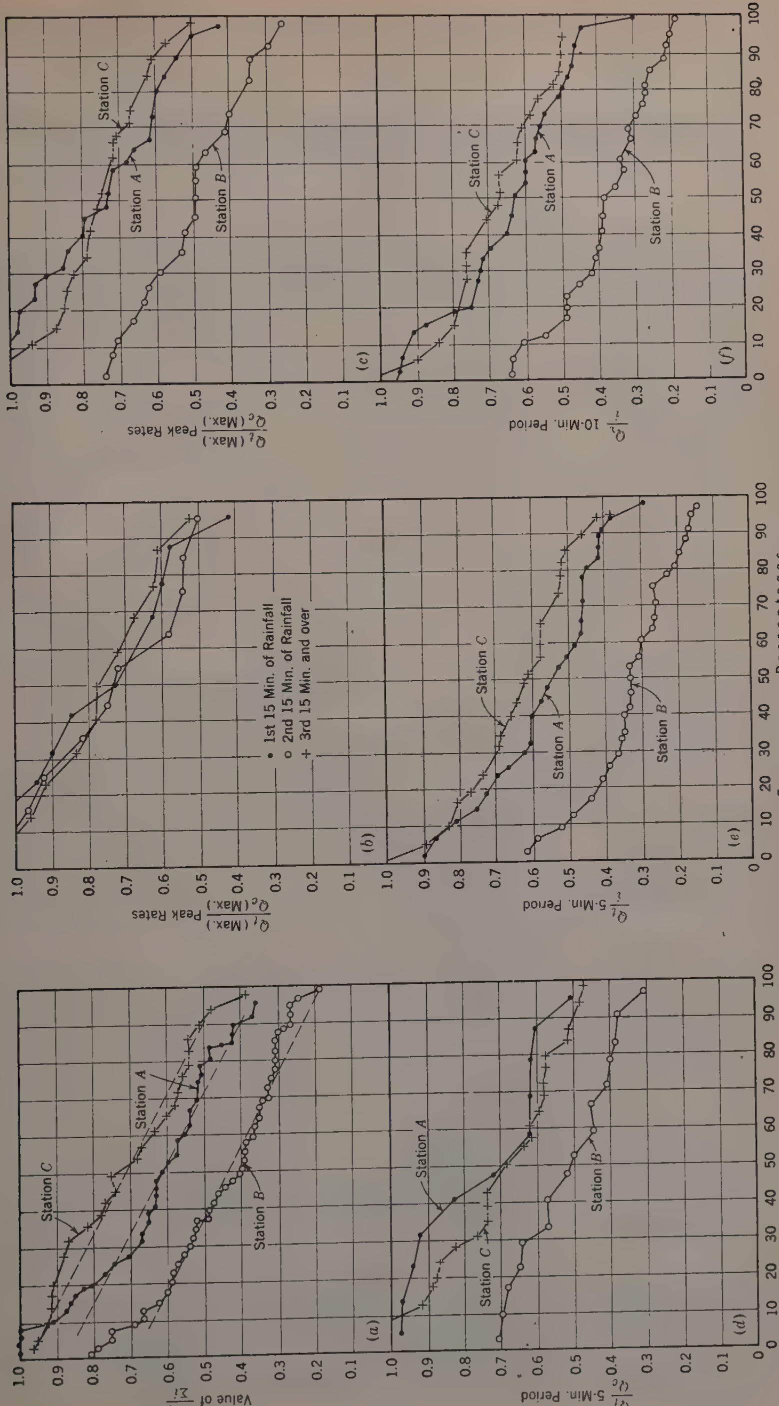


FIG. 18.—PERCENTILE CURVES OF RAINFALL-RUN-OFF RATIOS.

TABLE 4.—DATA USED IN FREQUENCY STUDY OF RAINFALL AND RUN-OFF RATES

Date	Rainfall							Run-Off							Run-Off / Rainfall							ΣQ / Σi
	Duration Period, in Minutes							Duration Period, in Minutes							Duration Period, in Minutes							
	5	10	15	20	30	40	60	5	10	15	20	30	40	60	5	10	15	20	30	40	60	
(a) STATION A, BELT AND RIDGE AVENUES																						
9/15/1914	3.12	2.91	(13) 2.80	(11) 2.47	(7) 2.40	(6) 2.16	(3) 1.77	1.89	1.57	1.41	(17) 1.47	1.4	(9) 1.21	(5) 0.95	0.61	0.54	0.50	0.60	0.59	0.56	0.54	0.58
5/ 2/1915	(18) 4.08	3.06	2.40	2.21	(13) 1.78	(16) 1.23	(19) 0.84	2.14	1.84	(18) 1.61	1.35	(14) 1.12	0.87	(17) 0.64	0.52	0.60	0.67	0.61	0.63	0.71	0.76	0.66
6/20/1915	3.24	2.93	(19) 2.64	(19) 2.27	(14) 1.74	....	....	1.53	1.40	1.24	1.09	0.83	(17) 0.89	(12) 0.73	0.47	0.48	0.47	0.48	0.48	....	....	0.46
6/20/1915	(15) 4.44	(12) 3.44	(14) 2.80	2.23	(15) 1.71	....	....	1.62	1.43	1.17	0.93	....	....	....	0.36	0.42	0.42	0.42	....	....	....	0.46
6/27/1915	2.88	2.76	2.42	2.22	(18) 1.65	....	....	1.36	1.31	1.19	1.07	0.81	....	....	0.47	0.47	0.49	0.48	0.49	....	....	0.51
8/ 2/1915	(12) 5.28	(10) 3.72	(16) 2.72	(12) 2.46	(19) 1.50	(17) 1.17	....	(9) 2.81	(11) 2.46	(9) 2.12	(9) 1.81	(10) 1.28	....	....	0.53	0.66	0.78	0.74	0.85	....	....	0.75
6/ 2/1916	(17) 4.20	(15) 3.40	(9) 3.28	(7) 2.87	....	....	....	2.06	(18) 1.93	(15) 1.96	(11) 1.80	(8) 1.35	....	....	0.49	0.57	0.60	0.63	....	....	....	0.70
8/ 2/1916	(4) 6.12	(8) 4.26	(8) 3.32	(8) 2.76	....	....	....	2.00	1.58	1.24	1.05	0.74	....	....	0.33	0.37	0.37	0.38	....	....	....	0.40
8/11/1916	(5) 6.12	(3) 4.86	(3) 3.68	....	....	....	....	(12) 2.74	(12) 2.36	(13) 2.00	(14) 1.62	....	....	....	0.45	0.49	0.54	....	....	....	....	0.58
8/12/1916	3.00	2.58	2.44	(18) 2.31	(12) 1.80	....	....	1.23	1.20	1.09	0.99	0.81	....	....	0.41	0.47	0.45	0.43	0.45	....	....	0.49
8/14/1916	3.78	2.94	2.28	2.10	(17) 1.68	....	....	1.93	1.74	1.37	1.16	(19) 1.00	0.80	....	0.51	0.59	0.60	0.55	0.60	....	....	0.65
8/14/1916	2.04	1.92	1.64	1.50	1.26	1.08	(18) 0.85	1.34	1.27	1.14	1.01	0.85	0.75	0.53	0.66	0.66	0.70	0.67	0.67	0.69	0.62	0.56
7/27/1917	....	....	....	....	....	(3) 2.36	(5) 1.58	....	....	....	....	....	(8) 1.29	(8) 0.88	....	....	....	....	....	0.55	0.56	....
9/28/1919	....	....	....	....	....	(6) 2.25	(2) 2.04	....	....	....	....	....	(12) 1.00	(7) 0.89	....	....	....	....	....	0.44	0.44	....
4/19/1920	(13) 5.04	(17) 3.27	2.56	2.03	(8) 2.25	(10) 1.61	(11) 1.15	1.71	1.44	1.27	1.13	(17) 1.06	(14) 0.95	(14) 0.69	0.34	0.44	0.50	0.56	0.47	0.59	0.60	0.57
9/ 8/1920	3.73	(16) 3.30	2.58	(10) 2 55	....	....	....	1.66	1.53	....	....	....	....	....	0.45	0.46	....	....	....	....	....	0.58
9/ 8/1920	(9) 5.64	(9) 4.26	(7) 3.53	(5) 3.09	(2) 2.94	(4) 2.30	(6) 1.53	(16) 2.50	(13) 2.22	(11) 2.01	(10) 1.81	(6) 1.65	(7) 1.34	(6) 0.90	0.44	0.52	0.57	0.59	0.56	0.58	0.59	0.58
9/11/1920	....	....	....	....	....	1.11	(15) 1.00	....	....	....	....	....	0.78	(16) 0.66	....	....	....	....	....	0.70	0.66	....
4/25/1921	(14) 5.04	(14) 3.42	(18) 2.67	(16) 2.34	(10) 1.91	(11) 1.56	(10) 1.22	(5) 3.03	(7) 2.59	(8) 2.17	(8) 1.95	(7) 1.63	(6) 1.42	(3) 1.07	0.60	0.76	0.81	0.83	0.85	0.91	0.88	0.93
4/26/1921	1.98	1.74	1.48	1.17	....	....	....	1.53	1.42	1.28	1.14	0.94	0.78	0.53	0.77	0.82	0.86	0.97	....	....	....	....
6/27/1921	(7) 5.88	(18) 3.27	2.20	....	....	....	....	(4) 3.12	(5) 2.69	(10) 2.07	(15) 1.61	(16) 1.10	....	....	0.53	0.82	0.94	....	....	....	....	1.00+
8/27/1921	(8) 5.76	(6) 4.38	(2) 3.86	(2) 3.53	(3) 2.92	(2) 2.43	(4) 1.71	(2) 3.66	(2) 3.19	(2) 3.00	(2) 2.68	(2) 2.26	(2) 1.90	(2) 1.33	0.64	0.73	0.78	0.76	0.77	0.78	0.78	0.86
4/14/1922	(6) 5.91	3.00	....	....	....	....	....	(10) 2.80	(15) 2.20	(19) 1.61	1.25	0.86	....	....	0.47	0.73	....	....	....	....	....	0.86
8/22/1922	(2) 6.48	(7) 4.38	(11) 3.05	(17) 2.34	1.44	(14) 1.32	(17) 0.89	(8) 2.81	(8) 2.57	(12) 2.01	(13) 1.63	(12) 1.17	(18) 0.89	....	0.43	0.59	0.66	0.70	0.81	0.67	....	0.76
8/ 8/1923	(1) 7.20	(1) 5.94	(1) 5.48	(1) 5.10	(1) 4.54	(1) 3.09	(1) 2.24	(3) 3.19	(3) 3.05	(3) 2.94	(1) 2.81	(1) 2.58	(1) 2.08	(1) 1.45	0.44	0.51	0.54	0.55	0.57	0.67	0.65	0.59
6/18/1924	2.16	1.56	1.64	1.77	1.34	1.05	....	1.48	1.33	1.18	1.12	0.90	0.72	....	0.69	0.85	0.72	0.63	0.67	0.69	..--	0.59
8/24/1924	(19) 3.87	(11) 3.72	(10) 3.16	(9) 2.61	....	(12) 1.55	(13) 1.06	1.97	(19) 1.91	(16) 1.77	(16) 1.53	(13) 1.13	0.87	....	0.51	0.51	0.56	0.59	....	0.56	....	0.65
6/28/1925	3.64	2.76	2.22	1.65	1.40	1.06	0.77	(15) 2.54	1.88	1.42	1.17	0.83	....	....	0.70	0.68	0.64	0.71	0.71	....	....	0.67
8/11/1925	(16) 4.32	2.47	1.72	1.34	1.21	1.08	0.75	(18) 2.27	1.48	1.14	0.85	....	....	....	0.53	0.60	0.66	0.63	....	....	....	0.60
9/12/1925	3.12	2.76	2.60	2.19	....	....	....	(11) 2.80	(6) 2.65	(6) 2.31	(6) 2.04	....	....	....	0.90	0.96	0.89	0.93	....	....	....	....
9/12/1925	3.72	(13) 3.42	(12) 2.83	(13) 2.42	(4) 2.81	....	....	(6) 2.97	(4) 3.01	(4) 2.67	(4) 2.32	....	....	....	0.80	0.88	0.94	0.96	....	....	....	....
5/19/1926	3.84	2.64	2.08	1.66	1.41	....	....	(19) 2.25	(16) 2.04	(17) 1.76	(18) 1.41	(15) 1.11	(16) 0.89	....	0.59	0.77	0.85	0.85	0.79	....	....	0.84
8/23/1926	(11) 5.28	(4) 4.47	(4) 3.64	(6) 3.05	(9) 2.20	....	....	(1) 3.74	(1) 3.37	(1) 3.05	(3) 2.63	(3) 1.95	(3) 1.52	....	0.71	0.75	0.84	0.86	0.89	....	....	1.00+
8/31/1926	3.76	2.57	1.80	....	....	....	....	(13) 2.69	1.91	1.45	1.13	0.77	....	....	0.72	0.74	0.81	....	....	....	....	0.86
9/ 8/1926	(3) 6.36	(5) 4.38	(6) 3.55	(4) 3.09	(5) 2.56	(8) 1.97	(7) 1.52	(7) 2.97	(10) 2.52	(7) 2.27	(7) 2.02	(5) 1.73	(4) 1.45	(13) 0.69	0.47	0.57	0.64	0.65	0.68	0.74	0.45	0.66
5/ 7/1927	3.16	(19) 3.12	(15) 2.75	(15) 2.35	(16) 1.70	....	....	1.57	1.53	1.42	1.36	0.77	0.82	(15) 0.67	0.50	0.49	0.52	0.58	0.45	....	....	0.61
5/28/1927	3.82	2.88	2.32	....	....	....	....	1.90	(17) 1.93	1.60	(19) 1.38	....	....	....	0.50	0.67	0.69	....	....	....	....	0.73

TABLE 4.—(*Continued*)

Date	Rainfall							Run-Off							Run-Off / Rainfall							$\frac{\Sigma Q}{\Sigma i}$
	Duration Period, in Minutes							Duration Period, in Minutes							Duration Period, in Minutes							
	5	10	15	20	30	40	60	5	10	15	20	30	40	60	5	10	15	20	30	40	60	
(a) STATION A, BELT AND RIDGE AVENUES (*Continued*)																						
6/19/1928	2.67	2.31	2.04	1.78	1.39	....	....	(17) 2.30	(14) 2.21	(14) 1.98	(12) 1.72	(9) 1.34	(10) 1.10	(10) 0.76	0.86	0.96	0.97	0.97	0.96	....	....	0.85
6/19/1928	3.48	3.12	(17) 2.68	(14) 2.40	(11) 1.90	(13) 1.53	(14) 1.05	(14) 2.61	(9) 2.53	(5) 2.35	(5) 2.18	(4) 1.76	(5) 1.42	(4) 0.99	0.75	0.81	0.88	0.91	0.93	0.93	0.94	......
7/ 5/1928	(10) 5.58	(2) 4.89	(5) 3.60	(3) 3.42	(6) 2.53	(7) 2.00	(8) 1.49	1.62	1.47	1.40	1.33	(11) 1.21	(11) 1.01	(11) 0.74	0.29	0.30	0.39	0.39	0.48	0.51	0.50	0.37
4/20/1929	....	....	....	....	....	(15) 1.32	(9) 1.23	....	....	....	....	....	(15) 0.94	(9) 0.84	....	....	....	....	....	0.71	0.68	......
5/18/1929	....	....	....	....	....	(18) 1.17	(16) 0.91	....	....	....	....	....	0.76	(18) 0.61	....	....	....	....	....	0.65	0.67	......
5/30/1929	....	....	....	....	....	(9) 1.95	....	....	....	....	....	....	(13) 0.95	....	....	....	....	....	....	0.49	....	......
9/14/1930	Imperfect rainfall record							....	....	....	....	....	(19) 0.88	(19) 0.61	....	....	....	....	....	....	....	......
9/ 1/1931	2.90	2.75	2.26	1.96	1.45	....	....	1.35	1.29	1.23	1.13	0.93	....	....	0.47	0.47	0.54	0.58	0.64	....	....	0.67
9/ 1/1931	1.87	1.69	1.56	1.50	1.36	(19) 1.13	(12) 1.08	1.26	1.23	1.19	1.16	(18) 1.03	Too small		0.67	0.73	0.76	0.77	0.76	....	....	......
(b) STATION B, CITY BLOCK No. 4841																						
9/ 5/1914	(7) 5.24	(11) 3.80	....	....	....	....	....	0.76	0.67	0.57	0.48	....	....	....	0.15	0.18	....	....	....	....	....	0.25
9/15/1914	(8) 5.08	2.97	2.50	2.07	(16) 1.63	(7) 2.06	(8) 1.56	0.97	0.92	0.86	0.80	0.72	0.64	(14) 0.50	0.17	0.31	0.34	0.39	0.44	0.31	0.32	0.35
6/13/1915	(2) 6.84	(2) 5.76	....	....	....	....	....	1.26	1.17	1.02	0.86	0.61	....	....	0.18	0.20	....	....	....	....	....	0.31
6/20/1915	(12) 4.75	(14) 3.52	(13) 2.77	(12) 2.33	(13) 1.77	(9) 1.94	(7) 1.62	(16) 1.29	1.17	0.93	0.75	0.60	(16) 0.66	(16) 0.49	0.27	0.33	0.34	0.32	0.34	0.34	0.30	0.35
6/27/1915	2.98	2.76	2.52	(16) 2.21	1.61	(15) 1.42	(14) 1.01	0.75	0.72	0.69	0.64	0.53	0.43	....	0.25	0.26	0.27	0.29	0.33	0.30	....	0.31
7/ 7/1915	3.12	2.82	(14) 2.73	(11) 2.56	(11) 1.91	....	....	1.12	0.94	0.95	0.92	0.75	0.59	....	0.36	0.33	0.35	0.36	0.39	....	....	0.36
8/ 2/1915	(11) 4.92	(7) 4.24	(4) 3.86	(8) 2.90	(8) 2.10	(16) 1.42	(16) 0.95	(15) 1.32	(16) 1.19	1.02	0.91	0.73	....	....	0.27	0.28	0.26	0.31	0.35	....	....	0.34
8/20/1915	....	....	....	....	....	0.75	0.72	....	....	....	....	....	....	0.46	....	....	....	....	....	....	0.64	......
5/28/1915	2.88	2.64	2.12	1.83	1.26	....	....	(14) 1.40	(14) 1.26	(14) 1.10	(14) 0.96	0.74	0.60	0.43	0.49	0.48	0.52	0.52	0.59	....	....	0.74
6/ 2/1916	3.96	3.18	(15) 2.68	(10) 2.58	....	....	....	(9) 1.55	(8) 1.51	(5) 1.47	(5) 1.35	(6) 1.09	(10) 0.88	(11) 0.61	0.44	0.48	0.55	0.52	....	....	....	0.57
8/11/1916	(15) 4.32	(13) 3.72	(10) 2.92	(13) 2.31	....	....	....	(10) 1.52	(12) 1.37	(13) 1.15	(13) 0.98	0.74	0.60	....	0.35	0.37	0.39	0.42	....	....	....	0.43
8/12/1916	2.40	2.04	2.07	1.95	(12) 1.90	(11) 1.82	(11) 1.30	1.28	(15) 1.26	(12) 1.25	(9) 1.22	(7) 1.08	(8) 0.96	(8) 0.70	0.53	0.62	0.61	0.63	0.57	0.53	0.54	0.60
8/14/1916	2.52	2.22	2.00	1.74	1.62	1.27	0.89	(13) 1.48	(11) 1.41	(11) 1.28	(11) 1.13	(10) 0.94	(12) 0.80	(12) 0.60	0.59	0.64	0.64	0.65	0.58	0.63	0.68	0.59
8/15/1916	....	....	....	....	....	1.27	0.86	....	....	....	....	....	(11) 0.85	(7) 0.74	....	....	....	....	....	0.67	0.86	......
9/ 7/1916	(6) 5.28	(8) 4.14	(11) 2.90	....	....	1.27	0.86	(5) 1.67	(5) 1.57	(9) 1.35	(12) 1.12	(12) 0.83	0.65	0.42	0.32	0.38	0.47	....	....	0.51	0.49	0.51
7/27/1917	(3) 6.00	(5) 4.63	(5) 3.72	(4) 3.47	(6) 2.70	(5) 2.40	(5) 1.98	1.06	(16) 1.03	1.00	(15) 0.95	(15) 0.80	0.64	....	0.18	0.22	0.27	0.27	0.30	0.32	....	0.30
8/24/1918	2.40	2.22	2.04	1.89	(15) 1.70	(14) 1.45	(13) 1.12	0.89	0.87	0.84	0.78	0.67	0.60	0.45	0.37	0.39	0.41	0.41	0.39	0.41	0.40	0.39
10/27/1918	4.20	(16) 3.42	2.48	....	....	....	....	0.92	0.86	0.73	0.60	0.43	0.33	....	0.22	0.25	0.29	....	....	....	....	......
9/28/1919	(14) 4.56	(9) 4.14	(7) 3.50	(3) 3.48	(3) 3.01	(4) 2.61	(4) 2.34	1.10	1.07	1.01	(16) 0.94	(13) 0.83	(13) 0.75	(10) 0.67	0.34	0.26	0.29	0.27	0.28	0.29	0.29	0.29
9/28/1919	2.76	2.34	2.04	1.98	1.56	1.52	1.27	0.95	0.90	0.86	0.82	0.69	0.62	(15) 0.50	0.34	0.38	0.42	0.41	0.44	0.41	0.39	......
4/19/1920	(9) 5.04	(15) 3.48	(12) 2.80	(15) 2.23	(9) 2.06	(8) 2.03	(10) 1.41	(3) 1.74	(7) 1.55	(10) 1.34	(10) 1.14	(5) 1.10	(7) 1.08	(5) 0.80	0.34	0.45	0.48	0.49	0.53	0.53	0.57	0.62
9/ 8/1920	4.20	3.20	2.56	2.19	(14) 1.76	....	....	(6) 1.64	(6) 1.56	(8) 1.40	(8) 1.25	(9) 0.96	(14) 0.75	....	0.39	0.49	0.55	0.57	0.54	....	....	0.60
3/26/1921	....	....	....	....	....	(13) 1.53	....	....	....	....	....	....	(4) 1.17	....	....	....	....	....	....	0.77	....	......
4/25/1921	4.20	3.18	(16) 2.66	(14) 2.25	(10) 1.91	(12) 1.74	(12) 1.27	(2) 1.90	(2) 1.74	(3) 1.51	(6) 1.33	(8) 1.05	(9) 0.94	(9) 0.68	0.45	0.55	0.57	0.60	0.55	0.54	0.54	0.61
8/26/1921	1.98	1.74	1.48	1.32	1.03	....	....	1.24	1.13	1.03	0.93	(16) 0.75	0.64	....	0.63	0.65	0.70	0.70	0.73	....	....	0.79
8/27/1921	(13) 4.72	(10) 3.84	(6) 3.71	(6) 3.30	(4) 2.90	....	....	(7) 1.61	(4) 1.60	(2) 1.54	(2) 1.46	(2) 1.34	(3) 1.21	(4) 0.85	0.34	0.42	0.41	0.44	0.42	....	....	0.49

TABLE 4.—(*Continued*)

Date	Rainfall							Run-Off							Run-Off / Rainfall							$\frac{\Sigma Q}{\Sigma i}$
	Duration Period, in Minutes							Duration Period, in Minutes							Duration Period, in Minutes							
	5	10	15	20	30	40	60	5	10	15	20	30	40	60	5	10	15	20	30	40	60	
(*b*) STATION *B*, CITY BLOCK No. 4841 (*Continued*)																						
7/16/1923	(4) 5.52	(4) 4.68	(2) 4.01	(2) 3.66	(2) 3.43	(3) 3.35	(2) 2.44	(11) 1.51	(10) 1.47	(7) 1.42	(4) 1.39	(3) 1.33	(2) 1.24	(2) 0.90	0.27	0.31	0.35	0.38	0.39	0.37	0.37	0.36
8/ 8/1923	(1) 8.76	(1) 8.34	(1) 7.88	(1) 7.11	(1) 6.36	(1) 4.71	(1) 3.34	(1) 2.43	(1) 2.36	(1) 2.30	(1) 2.23	(1) 2.15	(1) 1.95	(1) 1.40	0.27	0.27	0.29	0.31	0.34	0.41	0.42	0.40
6/23/1924	(16) 4.32	(12) 3.78	(9) 3.32	(5) 3.42	(5) 2.80	(10) 1.94	(9) 1.44	(12) 1.50	(9) 1.47	(6) 1.46	(3) 1.40	(4) 1.26	(6) 1.10	(6) 0.80	0.35	0.39	0.44	0.41	0.45	0.57	0.55	0.53
8/24/1924	(5) 5.44	(3) 4.74	(3) 3.92	(7) 2.91	2.00	1.14	(15) 0.96	(4) 1.70	(3) 1.65	(4) 1.50	(7) 1.27	(11) 0.87	....	....	0.31	0.35	0.38	0.44	0.43	....	....	0.44
9/12/1925	....	....	....	....	....	(2) 3.36	(3) 2.43	....	....	....	....	....	(5) 1.16	(3) 0.88	....	....	....	....	....	0.35	0.36	......
9/ 8/1926	(10) 5.04	(6) 4.62	(8) 3.48	(9) 2.89	(7) 2.46	(6) 2.15	(6) 1.75	1.02	0.99	0.93	0.89	(14) 0.81	(15) 0.73	(13) 0.53	0.20	0.21	0.27	0.31	0.33	0.34	0.30	0.31
6/17/1928	3.84	3.12	2.32	1.83	....	....	....	(8) 1.61	(13) 1.29	(15) 1.04	0.85	0.39	....	....	0.42	0.41	0.45	0.47	....	....	....	0.48
(*c*) STATION *C*, EWING AND WASHINGTON AVENUES																						
8/11/1916	3.48	3.06	(11) 2.91	(12) 2.36	(11) 1.60	(9) 1.37	....	1.98	1.86	1.66	1.37	1.02	0.82	....	0.57	0.61	0.57	0.58	0.64	0.60	....	0.69
6/ 2/1916	3.72	(10) 3.36	(10) 2.92	(8) 2.53	(7) 1.82	....	....	1.94	1.70	1.53	1.33	1.00	0.80	....	0.52	0.51	0.52	0.53	0.55	....	....	0.57
9/ 7/1916	3.72	3.30	(8) 2.98	(10) 2.40	(9) 1.72	....	....	1.98	1.64	1.50	1.30	0.97	....	....	0.53	0.50	0.50	0.54	0.56	....	....	0.56
7/27/1917	(8) 4.80	(5) 3.96	(5) 3.36	(5) 2.79	(4) 2.18	(5) 1.65	1.12	2.22	(12) 2.10	(10) 1.80	(10) 1.58	(11) 1.20	(9) 0.98	(8) 0.69	0.46	0.53	0.54	0.57	0.55	0.59	0.62	0.59
8/ 5/1917	3.00	2.34	1.96	1.62	1.20	1.19	0.82	1.55	1.37	1.16	0.99	0.76	0.70	0.53	0.52	0.59	0.59	0.61	0.63	0.59	0.65	0.39
9/17/1917	(11) 4.56	(8) 3.42	(7) 3.00	(9) 2.43	....	....	....	(11) 2.62	(10) 2.17	(8) 1.90	(9) 1.60	(12) 1.17	....	....	0.57	0.63	0.63	0.66	....	....	....	0.74
8/24/1918	(3) 5.52	(2) 5.10	(1) 4.76	(1) 4.83	(1) 4.46	(1) 4.35	3.60	(1) 5.55	(1) 5.37	(1) 5.07	(1) 4.88	(1) 4.32	(1) 3.94	(1) 3.26	1.00	1.05	1.06	1.11	0.97	0.91	0.91	0.97
10/27/1918	(7) 4.92	(11) 3.36	2.26	1.70	1.20	....	....	(9) 3.20	(8) 2.38	(11) 1.78	1.42	1.04	(11) 0.88	....	0.65	0.71	0.79	0.84	0.87	....	....	0.93
6/17/1919	3.60	2.88	2.84	(6) 2.61	(5) 2.12	(4) 1.67	1.14	1.89	1.81	(12) 1.77	(7) 1.76	(5) 1.42	(4) 1.17	(4) 0.82	0.53	0.63	0.62	0.68	0.67	0.70	0.72	0.43
7/11/1919	(12) 4.56	(9) 3.42	(12) 2.88	(11) 2.37	(10) 1.64	(11) 1.26	0.88	1.97	1.73	1.60	(12) 1.44	1.09	(12) 0.86	(11) 0.61	0.43	0.51	0.56	0.61	0.67	0.68	0.69	0.74
9/21/1919	....	....	....	....	1.26	(12) 1.20	0.97	1.29	1.15	1.06	1.01	0.91	0.82	(7) 0.71	....	....	....	....	0.72	0.68	0.73	0.41
9/28/1919	(4) 5.52	(1) 5.34	(2) 4.52	(2) 3.90	(2) 3.12	(2) 2.94	2.92	1.58	1.48	1.40	1.32	(9) 1.29	(3) 1.29	(2) 1.27	0.29	0.28	0.31	0.34	0.41	0.44	0.44	0.21
9/ 8/1920	(6) 5.04	(7) 3.48	(6) 3.20	(7) 2.61	....	....	....	(5) 3.50	(6) 2.67	(4) 2.39	(4) 2.02	(4) 1.43	(7) 1.09	....	0.69	0.77	0.75	0.77	....	....	....	0.83
9/11/1920	2.04	1.86	1.88	1.71	1.30	1.11	0.85	1.13	1.06	1.00	0.89	0.73	0.58	0.41	0.55	0.57	0.53	0.52	0.56	0.52	0.48	0.60
4/25/1921	(1) 6.10	(4) 4.62	(4) 3.52	(4) 2.88	(3) 2.40	(3) 2.10	1.46	(3) 4.13	(3) 3.03	(5) 2.33	(2) 2.55	(2) 2.13	(2) 1.77	(3) 1.23	0.68	0.66	0.66	0.89	0.89	0.82	0.84	0.74
6/23/1924	2.64	2.10	1.80	1.79	1.50	(10) 1.30	1.26	1.60	1.39	1.20	1.08	0.84	....	....	0.61	0.66	0.67	0.72	0.56	....	....	0.49
8/24/1924	(2) 5.76	(3) 4.86	(3) 3.92	(3) 3.00	(6) 2.06	(6) 1.61	1.10	(2) 4.43	(2) 3.67	(2) 2.86	(3) 2.30	(3) 1.55	(5) 1.17	(6) 0.78	0.77	0.75	0.73	0.77	0.75	0.73	0.71	0.78
6/17/1925	3.96	3.00	2.32	1.89	....	....	....	(8) 3.23	(9) 2.32	(9) 1.82	(11) 1.46	(7) 1.35	....	....	0.81	0.77	0.78	0.77	....	....	....	0.76
6/28/1925	(9) 4.68	(12) 3.36	1.30	....	....	....	....	(6) 3.44	(5) 2.77	(6) 2.23	(6) 1.80	(10) 1.26	....	....	0.74	0.83	1.72	....	....	....	....	0.94
8/11/1925	(5) 5.40	(6) 3.54	(9) 2.96	2.25	(12) 1.54	(8) 1.40	1.05	(4) 3.54	(4) 2.79	(3) 2.39	(5) 1.97	(6) 1.35	(8) 1.08	(5) 0.82	0.66	0.79	0.81	0.88	0.88	0.77	0.77	0.90
9/ 8/1926	2.40	2.22	2.00	1.92	(8) 1.74	(7) 1.54	....	1.52	1.48	1.39	1.39	(8) 1.29	(6) 1.14	....	0.63	0.67	0.69	0.72	0.74	0.74	....	0.79
9/30/1926	2.40	1.80	1.55	1.38	1.20	1.12	0.75	1.68	1.44	1.32	1.16	1.02	0.72	(12) 0.58	0.70	0.77	0.85	0.84	0.85	0.64	0.77	0.89
10/ 2/1926	(10) 4.68	2.52	....	....	....	....	....	(10) 2.66	1.95	1.48	1.17	....	....	....	0.57	0.77	....	....	....	....	....	0.91
10/ 2/1926	3.84	3.24	2.32	1.80	1.24	0.95	0.64	(7) 3.39	(7) 2.57	(7) 2.08	(8) 1.66	1.16	(10) 0.90	(10) 0.63	0.88	0.79	0.90	0.92	0.94	0.95	0.98	0.96
5/31/1927	2.88	2.34	1.85	1.46	....	....	....	(12) 2.40	(11) 2.13	1.68	1.40	1.04	0.82	(9) 0.68	0.84	0.91	0.68	0.96	....	....	....	More than 100%

Relation of the Final Value of P to Other Variables.—In Fig. 20 an attempt is made to correlate the value of $P_1 = \frac{\Sigma Q_t}{\Sigma i}$ with the various factors indicated, and the only apparent correlation is in the case of the maximum

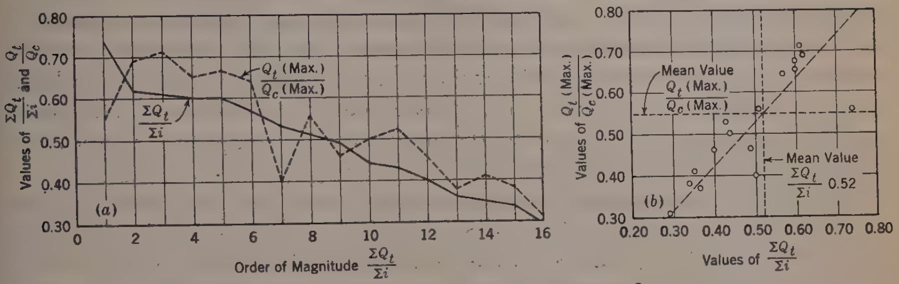


FIG. 19.—STATION *B*; CORRELATION DIAGRAM OF $\frac{\Sigma Q_t}{\Sigma i}$ AND $\frac{Q_{t\,(max)}}{Q_{c\,(max)}}$ FOR A 5-MINUTE DURATION PERIOD (BASED ON $j = 1$ AND $k = 1.2$).

run-off rate for a 5-min period. This indicates that high rates of run-off are more likely to be the result of a combination of relatively low rainfall rates and high run-off factors than the result of high rates of rainfall. Similar graphs for the other locations show similar results.

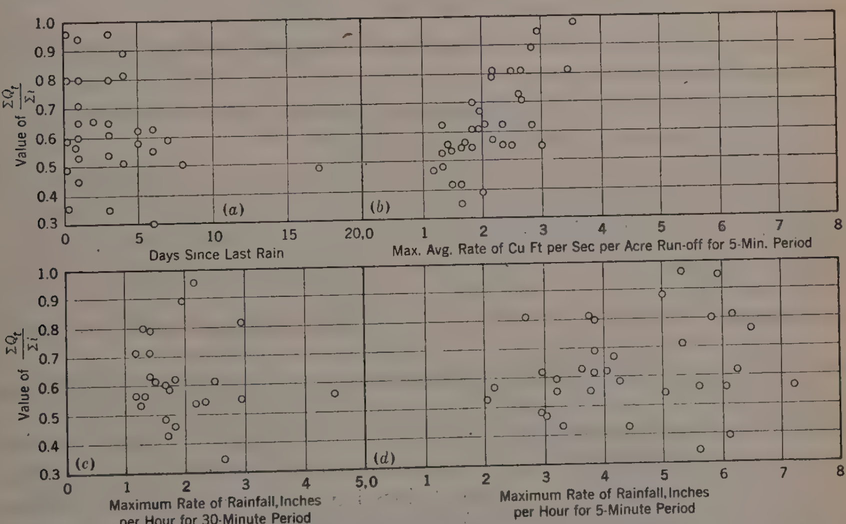


FIG. 20.—CORRELATION DIAGRAMS FOR $\frac{\Sigma Q_t}{\Sigma i}$; STATION *A*.

Ratio of Average Run-Off to Average Rainfall.—Pertinent information has been taken from the records and is given in Table 4. By means of these records it is possible to compare average rainfall rates with average run-off rates of the same duration, and also to compare these ratios with an end value of the run-off-to-rainfall ratio, P_1, as given in the last column.

In using Table 4, in the present case and in the frequency study that follows, it should be understood that the duration periods listed do not occur in the order listed for any particular storm; for example, at Station *A*, Table 4(*a*), for the rain of September 15, 1914, the entries under "20 minutes", for rainfall and run-off, which have numbers in parentheses of (11) and (17), are, respectively, the heaviest, average 20-min rainfall and the average rate of run-off for 20 min corresponding approximately with the particular rainfall occurrence. It is shown that the ratio of these average rates is 0.60 which corresponds closely to the ratio of the mean value for this storm. The numbers in parentheses correspond to the positions of these values in the frequency studies described subsequently.

Ratios of average run-off to average rainfall rates for 5-min duration and for 10-min duration have been taken from Table 4. As these values also vary over a wide range, they have been plotted as percentile curves, Fig. 18(*e*) and Fig. 18(*f*), and may be compared with other diagrams in Fig. 18. It should be noted that, while values on the various curves of Fig. 18 differ materially with the character of the ratio, the distribution of these values seems to follow about the same system for each diagram, as is indicated by a near parallelism of the lines. The close relationship of actual ratio values of curves in Fig. 18(*e*) and Fig. 18(*f*) with those of Fig. 18(*a*) is important.

General Conclusions to Part I

Certain general conclusions, which the writers are inclined to draw from these exhibits are, as follows:

(1) The value of all run-off ratios studied for individual storms varies between wide limits at each observation station.

(2) The values of "P_1 (final)" have been plotted as percentile curves for each of the areas under study. This curve, given as Fig. 18(*a*), is similar to Fig. 18(*c*), Fig. 18(*d*), Fig. 18(*e*), and Fig. 18(*f*), and shows that for each of the areas studied, P varies quite widely from the mean; but, as with the other factors, $\frac{Q_{t\,(max)}}{Q_{c\,(max)}}$ for absolute peaks; $\frac{Q_t}{Q_c}$ for 5-min averages at peaks; and $\frac{Q_t}{i}$ for 5-min and 10-min averages at peaks), the trend of the three curves is surprisingly uniform in character, indicating that the variables which cause this divergence enter into the occurrence for each of the drainage areas in about the same way. Straight lines have been drawn in Fig. 18(*a*) to represent an average of the percentile points. These lines indicate that the run-off at Station *A* throughout the full range of values is likely to exceed that at Station *B* by about 20%, while the rate for Station *C* will exceed that at Station *A* by about 10 per cent.

(3) For a given drainage area, the run-off factor, *P*, seems to be affected by the season of the year, general climatic conditions, previous precipitation, etc., but no co-ordination has been determined.

(4) No evidence of general correlation has been found between the "final value" of the factor, $P_1 = \frac{\Sigma Q_t}{\Sigma i}$, for individual storms, and the rainfall characteristics of the storms (such as maximum i or mean i). However, the variation of the run-off factor during the progress of a storm seems to depend, to some extent, upon the variation of the rainfall intensity. The effect is not pronounced except in extreme cases, such as double or multiple rains, when the rainfall intensity falls to low values for considerable periods between the peaks.

(5) The ratio, $\frac{\Sigma Q_t}{\Sigma i}$, or total measured volume of run-off to total measured volume of rainfall, has been found to have a close correlation with the ratio, $\frac{Q_{t\,(max)}}{Q_{c\,(max)}}$, based on 5-min duration periods (see Fig. 19, for example). The ratio, $\frac{\Sigma Q_t}{\Sigma i}$, can be found conveniently with great accuracy, and provides the best information as to the general relation between rainfall and consequent run-off rates for any given storm.

(6) No doubt, the value of the run-off factor, P, is greatly affected by the nature of the soil, but the local records can throw no light on this phase of the subject, since the soil is the same yellow clay at each of the observation stations. The sprinkling experiments were undertaken to fix the values given in this paper to a definite soil condition. (See Table 8.)

(7) The wide variation in the "final values" of P at each of the sites must be attributed to combinations of three variables: (*a*) Condition of the soil; (*b*) condition attributed to coverage; and (*c*) character of the distribution of the rainfall rates.

(*a*) Variation in Soil Condition.—This variation probably is closely similar to that of the soil-moisture content, and accordingly is affected by previous precipitation and by the seasons of the year.

(*b*) Variation in Condition of Coverage.—The effect of the character of the turf on the lawns and of the foliage on trees and shrubbery, also would appear to have some relation to the season of the year.

(*c*) Character of the Distribution of the Rainfall Rates.—The final value, P (which is the ratio of total run-off to total rainfall), is not subjected to the effect of distribution which the unit-graph study brings to the values of $\frac{Q_t}{Q_c}$ during the rainfall. Accordingly, it will vary seriously with the manner in which precipitation departs, in one way or another, from the condition of uniform intensity.

Summary of Conclusions to Part I

In Part I, studies have been made of the run-off factor under the unit-graph method in three forms: First, as the ratio, $\frac{Q_t}{Q_c}$, or instantaneous

ratios; second, as the ratio, $\frac{Q_t}{Q_c}$, for generally coincident peak values; and, third, as $P = \frac{\Sigma Q_t}{\Sigma Q_c}$, a moving ratio of mass values.

Studies have been made of the direct relation between run-off and rainfall: First, as ratios of average run-off rates to average rainfall rates for the same duration; and, second, as the final value, $P_1 = \frac{\Sigma Q_t}{\Sigma i}$. These experimentally determined ratios have been found to vary over a wide range, and a satisfactory correlation to other variables has not been found.

The unit-graph process is probably the best yet developed for analyzing rainfall and run-off data of the kind presented in this paper. If some modification of it can be applied generally to run-off data hereafter available, it will be possible to study the remaining variables entering into the relationship between rainfall and run-off in a much simpler form than heretofore has been undertaken.

Further prosecution of the investigation undertaken herein, might appear to involve a series of studies. First, the unit-graph method would yield a factor similar to the coefficient of retardation, representing the ratios between $Q_{c(max)}$ as used in this section, and average rainfall intensity for particular duration periods classified as to frequency of occurrence. The second series would involve a determination of the values of $\frac{Q_t}{Q_c}$, a run-off factor, as has been attempted, with the proper relation to frequency of occurrence.

If such information were satisfactorily developed, the run-off rates would be determined by applying to average rainfall intensity of the proper frequency and duration, a coefficient of retardation, thus reducing it to a 100% run-off rate, and, thereafter, applying the value of $\frac{Q_t}{Q_c}$ to reduce to actual run-off rates.

Such an undertaking, however, becomes involved in numerous complications. It appears that the same results could be secured by a simpler means, although probably in a less scientific manner, if the information on rainfall and on run-off was made the subject of frequency-of-occurrence studies as if the two phenomena were independent.

PART II.—Frequency Study of Run-Off Rates

In Part II, run-off is considered as an independent phenomenon, and a frequency study is made of the average rates of run-off for various duration periods at each observation station, in order to determine the answer to the question, "How often in a given number of years will a given inlet area discharge storm water at a given rate for a given duration period?"

Frequency is expressed in years and has reference to the period of time during which an event of given magnitude will probably be equalled or ex-

ceeded one time. Table 4 shows the basic data used in the frequency study. A period of years was chosen for each location such that a continuous series of the best records for that location was available. At Station *A*, the nineteen years, from 1914 to 1932, inclusive; at Station *C*, the twelve years, from 1916 to 1927, inclusive; at Station *B*, the sixteen years, from 1914 to 1929, inclusive; and at the United States Weather Bureau Station, the twenty-two years, from 1907 to 1930, inclusive, were chosen as the most representative. (On September 29, 1925, the rain gauge at Station *B* was taken out of service. After that date, rainfall rates for this location were obtained by interpolation from three near-by gauges.)

These series although shorter than might be desired, are longer than the usually accepted minimum of about ten years. At each station all storms having significant average run-off rates (about 1.0 cu ft per sec per acre, or more) for the duration periods represented in Table 4, are listed in chronological order.

Under the heading, "Rainfall", in Table 4, is listed the highest average rates of rainfall, observed during the storm in question, for the various duration periods. Under the heading, "Run-Off," is listed the highest average rates of run-off observed during the storm in question, for the various duration periods of run-off.

The numbers in parentheses, shown with the rates, represent the order in magnitude of that rate for the duration period in which it is shown. For example, referring to Table 4(*a*), during the storm of September 15, 1914, as observed at Station *A*, the highest average rainfall rate for a period of 20 min is 2.47 in per hr, and this rate for this duration period has been exceeded ten times in the nineteen years represented in the table. The highest average run-off rate, for a duration period of 20 min, is 1.47 cu ft per sec per acre, and this rate has been exceeded sixteen times during the nineteen years. Where no numbers in parentheses are shown, the rate has been exceeded more times than the number of years represented in the series.

The columns headed, $\frac{\text{Run-Off}}{\text{Rainfall}}$ (Table 4), contain the ratios of the run-off rates to the rainfall rates, as shown in the preceding columns. Of course, the duration periods shown for run-off rates are not identical in absolute time with the corresponding duration periods shown for rainfall rates, the difference being due to the lag. In one or two instances, the duration periods compared belong to entirely different phases of the rain.

The column headed, $\frac{\Sigma Q}{\Sigma i}$ (Table 4), shows the ratio of the total run-off to the total rainfall for the entire duration of the storm in question. This information is not given for a few storms of long duration where the rates are insignificant for most of the time, only the peak values being investigated.

Table 5, being taken from the records of the U. S. Weather Bureau, shows rainfall rates only. The information on which the frequency series was determined is complete and consecutive with the exception of one storm, that

TABLE 5.—DATA USED IN FREQUENCY STUDY OF RAINFALL
(U. S. WEATHER BUREAU DATA)

Date	Rainfall Intensities, for the Following Duration Periods (in Minutes):						
	5	10	15	20	30	40	60
August 7, 1907	5.40 (6)	5.04 (1)	4.24 (2)	3.48 (5)	2.78 (4)	2.27 (6)	1.59 (7)
July 6, 1908	4.56	4.26 (8)	3.68 (6)	3.21 (6)	2.18 (10)	1.63 (13)	....
July 10, 1909	....	....	....	....	1.60	1.44	1.37 (10)
September 4, 1910	3.72	3.18	2.96	2.67 (14)	2.14 (12)	1.82 (8)	1.29 (12)
September 4, 1911	3.12	2.40	2.24	2.04	1.68	1.47	1.29 (11)
June 5, 1912	....	....	1.64	1.35	1.34	1.16	1.29 (13)
July 14, 1912	4.56	4.44 (6)	3.92 (5)	3.60 (3)	3.34 (1)	3.24 (1)	2.96 (1)
July 1, 1913	4.44	4.02 (12)	3.12	2.49	1.82	1.50 (14)	1.08
August 19, 1914	3.12	2.88	2.68	2.64	1.92 (13)	1.49	....
September 1, 1914	5.28 (7)	4.68 (4)	4.08 (3)	3.75 (2)	3.06 (3)	2.67 (4)	1.86 (5)
September 15, 1914	....	....	2.08	2.13	1.90	1.79 (10)	1.49 (9)
June 20, 1915	3.24	3.18	2.64	2.16	1.52	1.67 (12)	1.55 (8)
August 11, 1916	4.32	4.08 (10)	3.52 (8)	2.67	1.82	1.30	....
August 12, 1916	5.28 (8)	4.74 (3)	4.32 (1)	3.81 (1)	3.10 (2)	2.79 (2)	1.87 (4)
August 14, 1916	5.64 (4)	4.38 (7)	3.36 (11)	2.97 (8)	2.74 (5)	2.70 (3)	2.03 (3)
September 27, 1916	6.72 (1)	4.98 (2)	3.40 (9)	2.70 (13)	1.92 (14)	1.46	....
October 30, 1916	4.92 (11)	3.06	....	....	....	....	....
July 23, 1917	6.12 (2)	4.56 (5)	4.00 (4)	3.60 (4)	2.58 (6)	....	....
September 7, 1917	5.52 (5)	4.02 (13)	3.40 (10)	2.82 (9)	1.88	1.50	0.99
August 24, 1918	3.84	3.48	2.96	2.73 (11)	2.22 (9)	2.09 (7)	1.80 (6)
May 4, 1919	5.64 (3)	4.08 (11)	2.88	2.25	1.56	1.19	....
June 17, 1919	4.20	3.48	3.68 (7)	3.21 (7)	2.34 (8)	1.80 (9)	1.21 (14)
June 22, 1919	4.80 (12)	3.12	2.32	....	....	....	....
September 28, 1919	3.48	3.42	3.12	2.82 (10)	2.50 (7)	2.58 (5)	2.40 (2)
August 15, 1920	4.80 (13)	3.96	3.20 (13)	2.55	1.78	1.36	....
September 8, 1920	5.04 (10)	3.96 (14)	3.28 (10)	2.52	....	....	....
September 8, 1920	5.28 (9)	4.26 (9)	3.20 (14)	2.70 (12)	2.14 (11)	1.71 (11)	....
December 13, 1920	4.60 (14)	3.12	2.32	....	....	....	....

of August 8, 1923, for Station C, and is accurate within the limitations of the instruments used. The years, 1918 and 1932, are not represented in Table 4(a) for Station A, nor are the years, 1922 and 1929, in Table 4(b) for Station B, as no rains of sufficient magnitude occurred in these years to warrant inclusion. Several storms were included for which the data were open to suspicion on the grounds of non-conformity, such as that of September 12, 1925, at Station A, and the small rains showing more than 100% run-off, but for which no definite defect in the record could be found.

Derivation of Series.—A frequency series was made up for each vertical column under the headings, "Rainfall and Run-Off", in Table 4. The rates taken from each column were arranged in the order of magnitude beginning with the highest. Since the class of storms that occur, on an average, more than once each year are of little or no interest in design, the number of terms in each series was arbitrarily limited to the number of years represented in Table 4 from which it was taken. This makes the number of events equal to the number of years in the series, facilitating the computation of the time frequency.

The frequency percentages for each series were computed from the formula[4]:

$$F = \frac{2\,m - 1}{2\,n} \qquad (5)$$

[4] "Method of Least Squares", by Mansfield Merriman, M. Am. Soc. C. E.

in which, F = percentage of frequency; m = ordinal number of the item in the series (the maximum being No. 1); and, n = number of items in the series.

The rates of rainfall and run-off as selected for the various series were plotted against the corresponding frequency percentages on logarithmic probability paper (Fig. 21). Since direct comparisons were to be made

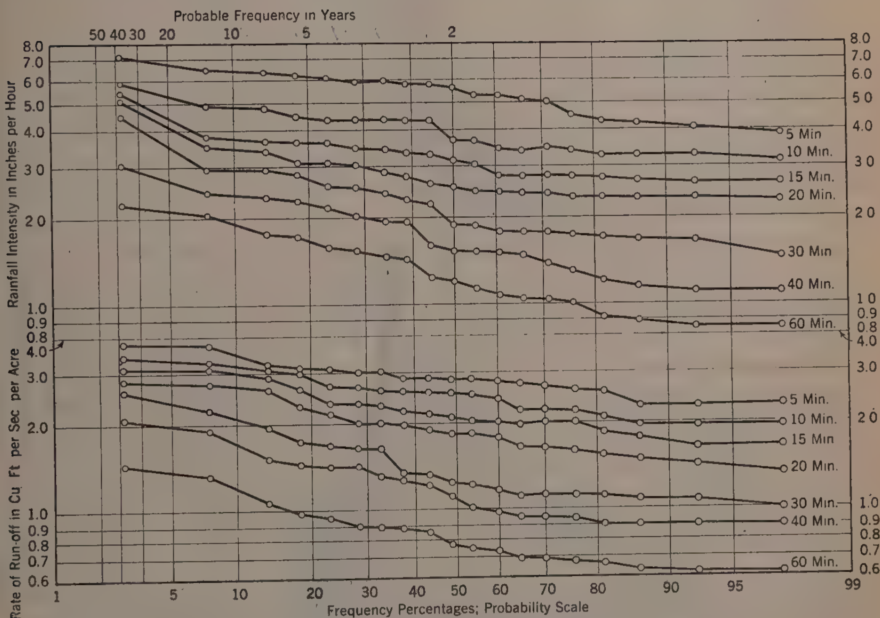


FIG. 21.—STATION *A*. RAINFALL AND RUN-OFF FREQUENCY SERIES, 1914 TO 1932, INCLUSIVE; RATE FREQUENCY CURVES.

between the different series, it was decided to derive the curves through the plotted points by computation, using statistical rather than graphical methods, in order to eliminate the personal equation and assure that each series would receive exactly the same treatment.

The method used in computing the points on the smooth curves in Fig. 21 consists of calculating: First, the mean value of the terms in each series; and, then, the coefficients of variation and skew for the series. Knowing the value of these coefficients, the co-ordinates of points on the skew frequency curve were calculated from the information given in the table[5] of skew curve factors by H. Alden Foster, M. Am. Soc. C. E. Details of these computations and complete sets of curves are filed with the record manuscript in Engineering Societies Library.

The curves, such as those shown in Fig. 21, indicate the probable frequency of occurrence of given rates of rainfall and run-off, expressed as a percentage of the total number of events of the same class in a given array;

[5] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), Table 2, p. 162.

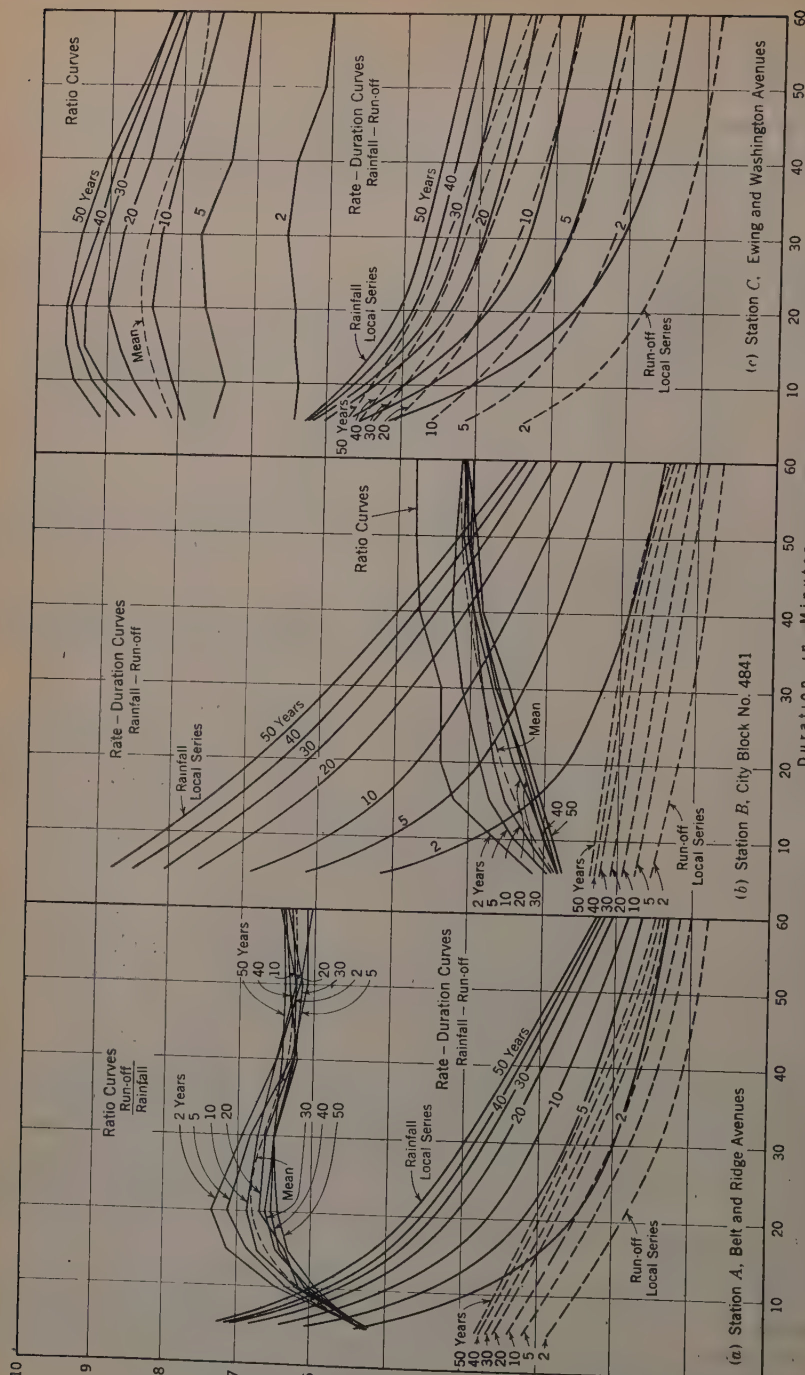


FIG. 22.—RELATION OF RATES OF RUN-OFF AND RAINFALL TO DURATION OF THESE RATES.

for example, a rainfall or run-off rate having an event frequency of 20%, as shown by the graph, will probably be equalled or exceeded once in five years.

Transposing the co-ordinate axes and plotting the data given by the curves of the type of Fig. 21, a series of curves are obtained which show the relation of the rates of run-off and rainfall to the duration of these rates for frequencies of 50, 40, 30, 20, 10, 5, and 2 years. These curves (Fig. 22) correspond to the familiar "rainfall-intensity" curve.

Correspondence Between Series.—There is no causal connection between corresponding points on the rainfall and the run-off curves for a given frequency. Such points do not represent actual storms, but only rates having the same probable frequency. The curves represent variations in concurrent, but not necessarily completely dependent, phenomena.

Fig. 22 shows also the ratios between corresponding rainfall and run-off rates for various frequencies taken from these rate-duration curves. The usefulness of these ratios lies in the fact that the data from several observation stations may be combined to provide a rainfall frequency series of greater length and possessing greater scope and accuracy than is possible in the case of the local rainfall and run-off frequency series.

Transformation to Long-Time Series.—The ratios between corresponding rainfall and run-off rates, as shown by the local rate-duration curves, reflect the local conditions that affect the rainfall-run-off relations at the station in question. Therefore, when these ratios are applied to rainfall rates given by the long-term rainfall frequency series, the resulting rate-duration curves should show the most probable values for run-off rates for various frequencies and duration periods at each station.

Another advantage of this method is that it takes into account the position factor of rainfall rates and its effect on run-off rates. A high rainfall rate at the beginning of a storm, may result in an insignificant rate of run-off while a much lower rate of rainfall of the same duration, occurring late in a storm, may result in a relatively high rate of run-off. In the long-term rainfall frequency series, all types of storms are included, and the position factor of the various rainfall rates is taken into account (at least indirectly) with the relative frequency of the various types of storms.

A long-time series, comprising 47 station-yr, may be obtained by combining the rainfall records of the three observation stations; but two of the stations (Stations *A* and *B*), are quite close together, while the third (Station *C*), is about four miles away (see Fig. 1). Since the U. S. Weather Bureau Station is fairly close to Station *C*, this fault may be minimized by combining the records from these four stations, thus obtaining a 61-station-yr series. It will be seen from Tables 1 to 5 that few critical storms are common to all four stations, and that the record at no one of the stations having a length of from 12 to 17 yr can be truly representative of the rainfall probabilities for the short storms of the St. Louis region. This 61-station-yr series, shown in Fig. 23(*b*), has been adopted as the basis of the final frequency study. It could have been improved somewhat by using all the rain-gauge sta-

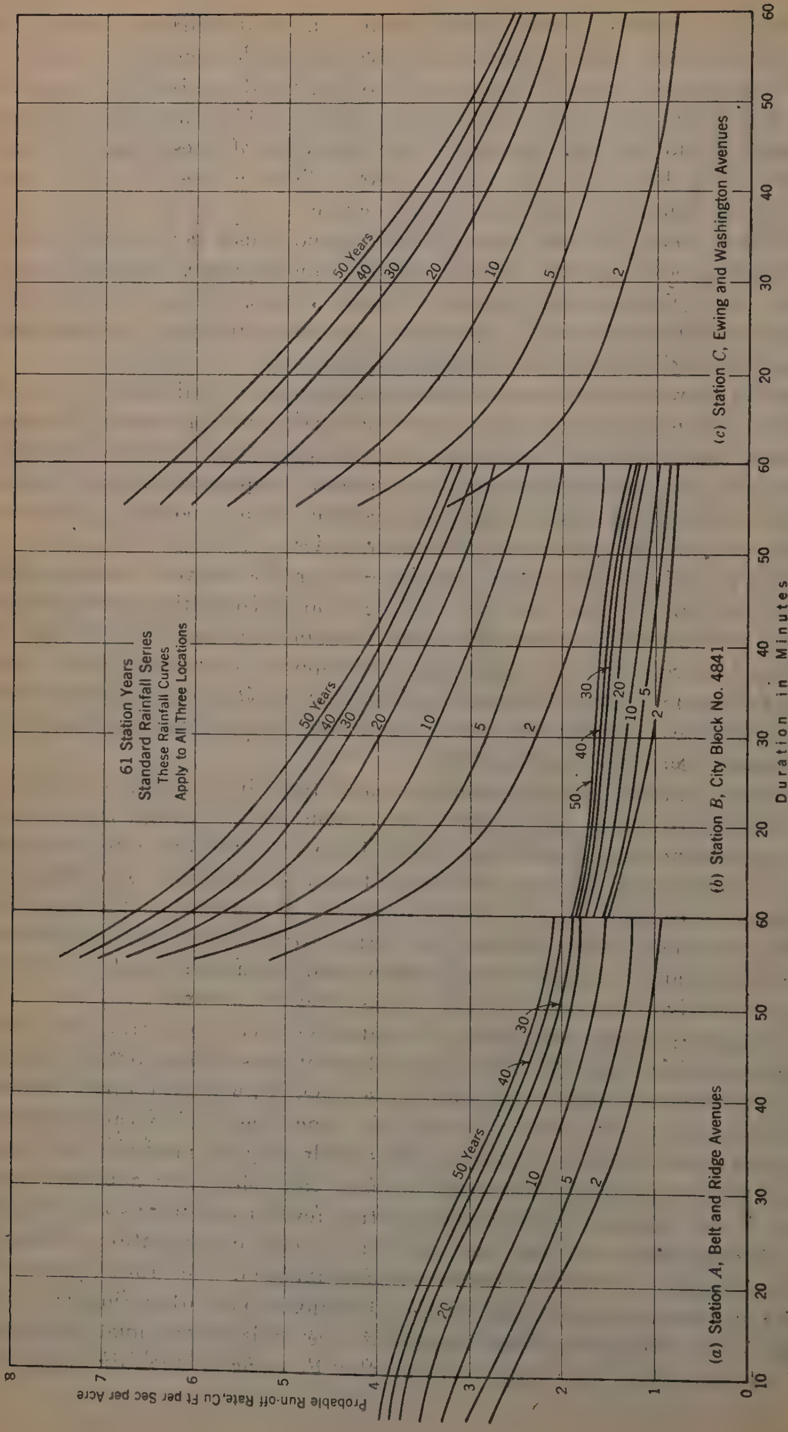


FIG. 23.—ADJUSTED RUN-OFF RATE CURVES BASED ON STANDARD RAINFALL SERIES.

tions in St. Louis, but this would have had the disadvantage of not involving to so great an extent the localities or the rainfalls entering into the run-off study.

For each of the three blocks studied, frequency curves were developed by applying to the 61-yr rainfall series the ratio between rainfall and run-off shown graphically in Fig. 22. Thus, the modified run-off frequency series as given on the lower part of Fig. 23 is assumed to represent equivalent probabilities with the 61-yr rainfall series.

To determine whether or not the basic information had been in any way distorted in its manipulation through the frequency studies, a second study was made in the form indicated in Part III.

Correlation Diagrams—Rainfall and Run-Off Rates for Various Duration Periods

The graphs in the series of curves represented by Fig. 24, of which only three are published, are compiled from two independent sets of data. The

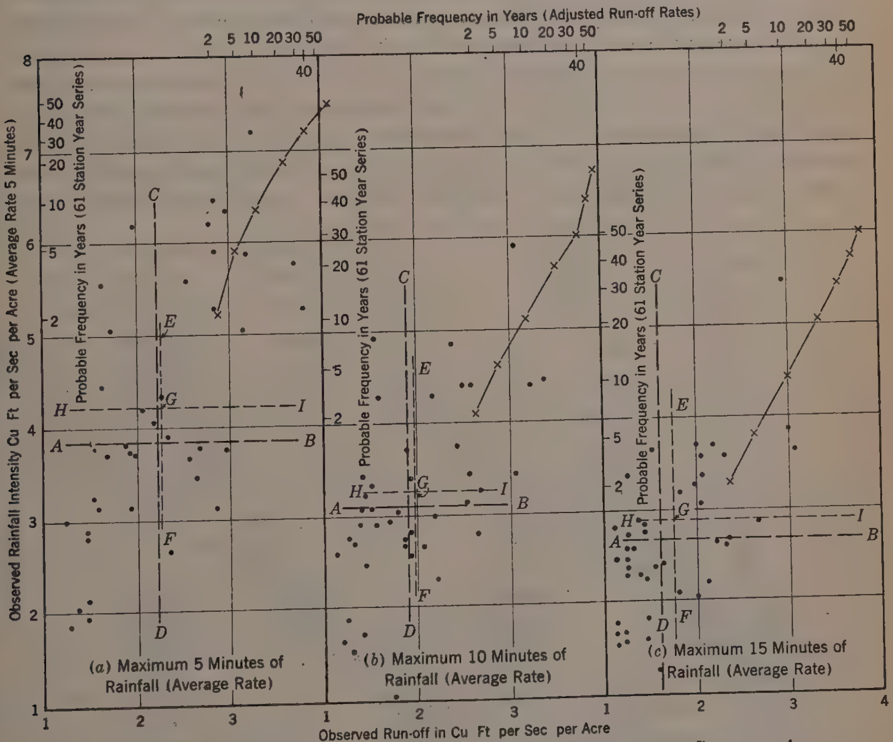


Fig. 24.—Correlation Curves of Rainfall and Run-Off, Station *A*.

plotted points on each graph represent the average rate of rainfall and run-off for the duration period for which it was prepared. Each point represents a single storm listed in the table from which the rainfall and run-off frequency series were compiled. Points above Line *A-B* (Fig. 24) represent storms included in the rainfall frequency series. Points to the right of Line *C-D*

represent storms included in the run-off frequency series. Lines *E-F* and *H-I* define the mean values of the rainfall and run-off rates in relation to the vertical and horizontal axes, respectively, thus establishing Point *G* as the center of gravity of all the points.

Referring to Table 4(*a*), it is seen that for the first storm listed at Station *A* (September 15, 1914), the maximum average rainfall rate for a 5-min period is 3.12 in. per hr, and the maximum average run-off rate for the same period is 1.89 cu ft per sec per acre. The point representing this storm, therefore, is located from these co-ordinate values on Fig. 24(*a*).

The frequency scales shown on the rainfall and run-off rate axes are taken from the rate-duration curves (see Figs. 23(*a*) and 23(*b*)) where it is found that the rainfall rate having a probable frequency of 2 yr (for 5-min duration period) is 5.10 in. per hr, while the corresponding run-off rate is 2.85 cu ft per sec per acre, etc., for each frequency.

The points designated by crosses in Fig. 23, and connected by solid lines, are plotted opposite corresponding frequencies as shown on the rainfall and run-off axes on the right-hand top margins of each chart. These points do not represent actual storms, but only abstract rainfall and run-off rates having the same probable frequency, in years, as determined by the rate-frequency graphs for Station *A*. The relation of this line to the points shows that, although points on the frequency curve were extrapolated beyond the observed data, the general trend of the ratio of run-off rate to rainfall rate as observed in the original data has been preserved in the extrapolated values.

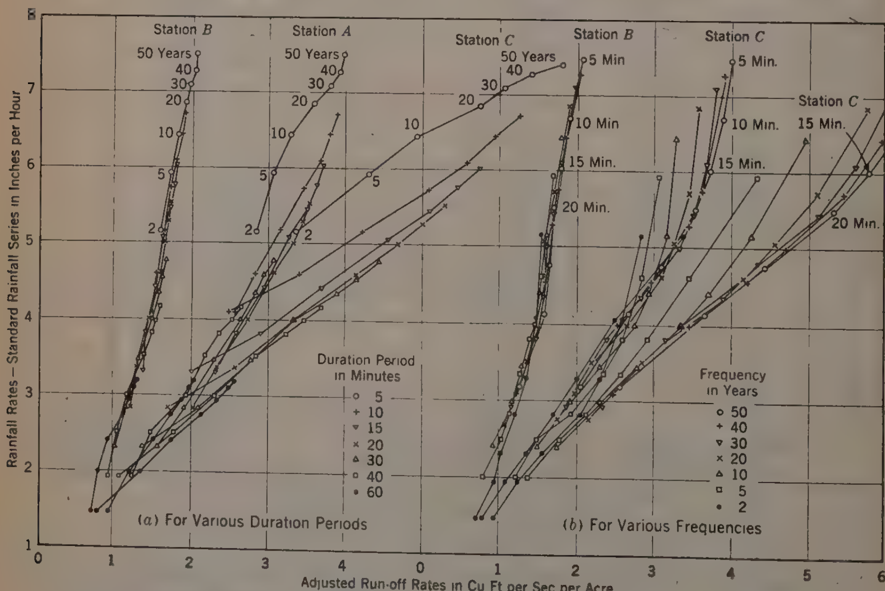


FIG. 25.—RELATION BETWEEN RAINFALL AND RUN-OFF RATES (SEE TABLE 6).

From this series of diagrams two composite charts (Fig. 25 and Table 6) were compiled, which gave identical information as to the plotted points,

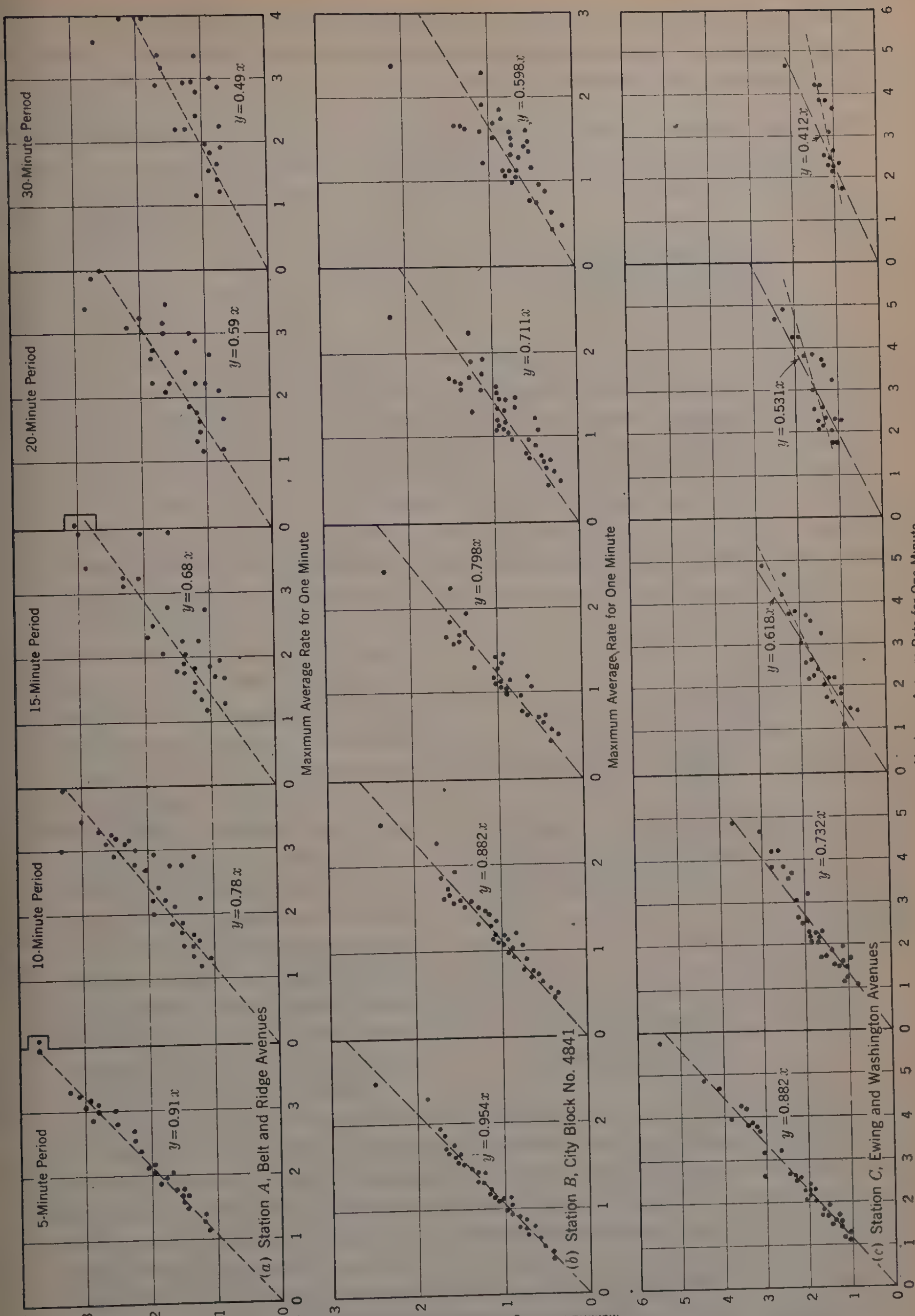


Fig. 26.—Correlation Diagrams; Maximum Average Run-Off Rates for Various Duration Periods Compared with Maximum Average Rate for 1 Minute (Units in Cubic Feet per Second per Acre)

and differed only in the connecting lines shown between the points. They involve essentially the same information that is presented in Fig. 23. Each of the diagrams offers different aspects.

TABLE 6.—RELATION BETWEEN RAINFALL AND RUN-OFF RATES FOR VARIOUS DURATION PERIODS AND FREQUENCIES (SEE FIG. 25)

Description	Station *A*	Station *B*	Station *C*
Tributary area, in acres	2.25	3.25	4.33
Percentage of area, impervious	50	29	72
Percentage of area, pervious	50	71	28
Average slope (percentages)	3	0.7	2.0

Figs. 23 and 25 contain the results of the run-off frequency study. However, in Fig. 26, this material has been set up to present one other relationship. These diagrams were prepared by comparing the average run-off rates for particular duration periods, as, for example, 5 min and 10 min, with the maximum average run-off rate for 1 min for the same storm. The information as to the average 5-min and 10-min rates was taken from Table 4. The tabulated data for the 1-min rate are not presented, but they were taken directly from the hydrographs. The relationships indicated in these diagrams, as summarized on Fig. 27, are surprisingly consistent. It should be noted that the relationship for Station *C* (Ewing and Washington Avenues) for the 20, 25, and 30-min periods, does not seem to follow a slope line through the origin as clearly as is the case for other curves, and the line drawn is a mean used only for the purpose of preparing the curve on Fig. 27. While it is not expected that the relations given by the curves can be of immediate use in connection with sewer, design, they are interesting and should be of real value to the hydrologist.

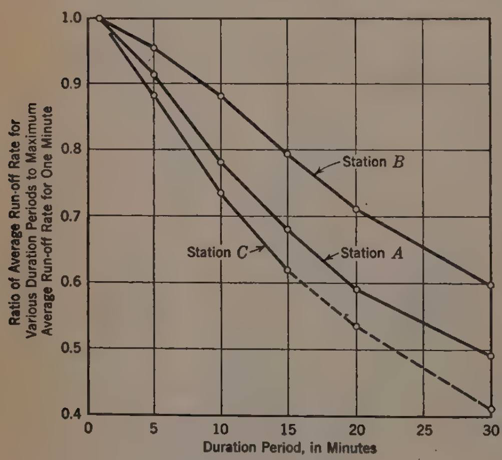


FIG. 27.

The run-off values shown on Fig. 22(*a*) for Station *A* and those for Station *B* in Fig. 22(*b*), seem to fit into a logical conception of the effect of topography and surface condition on run-off rates. The curve for Station *C* (Fig. 22(*c*)) giving the result for an entire city block, including many sub-areas such as roofs and yards that drain directly to the sewer, is not strictly comparable to the information for the other two blocks. The very high run-off rates shown in this case were surprising to the writers, even in view of their familiarity with conditions.

Figs. 22(*a*) and 22(*b*), show that for the two smaller locations, the greatest ratio between run-off and rainfall occurs for the lighter and more frequent

rains; for Area C the reverse is true. Tracing the reversal phase back into the primary data, it is noted that the corresponding rainfall and run-off curves for Area C diverge to the right, whereas for the other sites these curves approach each other for the longer time.

This means that the range of run-off rates at Station C is greater than the range of rainfall rates; that is, the drainage system at Station C is more sensitive to changes in rainfall rates than the systems at the other locations, due probably to the manner of collecting the water. The mean velocity from points on Area C is greater than that from points on Area A at the same distance from the point of measurement.

Fig. 25 brings out in an interesting manner the fact that the run-off characteristics of the three sites are quite similar for the light rainfalls, and it is only in the field of the rarer occurrences and greater intensities that the marked differences appear. For Station B, the relationship of rainfall and run-off is affected, to a relatively small degree, by variation in duration or in frequency; for Station A, more important differences appear on this account; and, for Station C, the effects are distinctive.

Conclusions to Part II

At various times during the period, 1924 to 1934, the writers and their associates have attempted to organize the information given herein in a manner that would justify publication. The present effort at analysis has extended over a period of nearly two years, and the writers now recognize, more fully than ever, the inadequacies of the results secured. They are more than ever impressed with the certainty that much of importance still lies buried in the available data. The paper in its present form is offered in the hope that a wider study, and the application of other minds, may bring in more constructive interpretations.

To the hydrologists, they believe that they are presenting hitherto unpublished information susceptible of detailed study and interpretation.

For the engineer designer in the municipal drainage field, the information contained in Part II should be a valuable guide to judgment, although it is only a small fragment of the information desirable for reaching a sound basis for storm-sewer design. In Part III a method is suggested in the application of which the information presented will be usable in a broader field without departing too far from the available facts and figures.

PART III.—Suggestions for Application to Sewer Design

A study of the diagrams, Figs. 23 and 25, brings out the following crude relationships:

(1) For flat areas of relatively low percentage of imperviousness (such as, Area B), the relation between rainfall and run-off of equal probability does not vary greatly with frequencies or durations; the average ratio is slightly less than 0.4

(2) For steeper and less permeable areas (such as Area A), the correlation is fairly uniform except for the shorter 5-min rains, and the average ratio is about 0.65.

(3) For the closely built-up block (such as Area *C*), the correlation is not so simple, and the average values are greater than 0.8.

(4) Table 7 gives in round figures the principal characteristics of the three test sections.

TABLE 7.—PRINCIPAL CHARACTERISTICS OF AREAS STUDIED

Area (see Fig. 16) (1)	Average slope (percentages) (2)	Percentage of area, impervious (3)	General average ratio of run-off rates to rainfall rates (4)
B.................	0.7	30	0.4
A.................	3.0	50	0.65
C.................	2.0	72	0.8

(5) With only three sites sufficient sets of information are not available to permit of a proper attempt to isolate the effect on the run-off ratio (Column (4) Table 7), of the variable in Column (2), from that of the variable in Column (3). It has been interesting, however, to attempt to set up a series of ratios for the pervious and impervious areas, found by "cut-and-try", that would develop the actual run-off rates given in the final diagrams.

A typical, although incomplete, study of this type for a 15-yr rain is presented in Table 8.

TABLE 8.—EFFECT OF GROUND SURFACE SLOPE ON THE RUN-OFF RATIO (COLUMN (4), TABLE 7) (FIFTEEN-YEAR STORM DATA)

Item No. (1)	Area (see Figs. 2, 3, and 4) (2)	Percentages of Area: Impervious (3a)	Percentages of Area: Pervious (3b)	Per centage of run-off, *P* (4)	Rainfall rate, in inches per hour (5)	Run-Off, in Units, Inches per Hour: Computed: Product of Columns (3), (4), and (5) (6)	Run-Off, Computed: Total (7)	Run-Off: Measured (8)
				(*a*) DURATION PERIOD, 50 MINUTES				
1...	*B*	30	....	80	2.80	0.67	1.16	1.15
2...	*B*	....	70	25	2.80	0.49		
3...	*A*	50	....	90	2.80	1.26	1.89	1.80
4...	*A*	....	50	45	2.80	0.63		
5...	*C*	72	....	90	2.80	1.81	2.28	2.20
6...	*C*	....	28	60	2.80	0.47		
				(*b*) DURATION PERIOD, 30 MINUTES				
7...	*B*	30	....	80	3.70	0.89	1.46	1.40
8...	*B*	....	70	22	3.70	0.57		
9...	*A*	50	....	90	3.70	1.67	2.41	2.45
10...	*A*	....	50	40	3.70	0.74		
11...	*C*	72	....	90	3.70	2.40	3.02	3.10
12...	*C*	....	28	60	3.70	0.62		
				(*c*) DURATION PERIOD, 20 MINUTES				
13...	*B*	30	....	75	4.3	0.97	1.57	1.50
14...	*B*	....	70	20	4.3	0.60		
15...	*A*	50	....	88	4.3	1.89	2.64	2.90
16...	*A*	....	50	35	4.3	0.75		
17...	*C*	72	....	88	4.3	2.72	3.39	3.70
18...	*C*	....	28	56	4.3	0.67		

Computation Table 8 is not offered as a good solution of this problem; the writers merely wish to leave it to practical sewer designers for further analysis. Note that the "pervious" surface on Areas *A* and *B* (Fig. 5(*a*)) is turf, and that on Area *C* (Fig. 5(*b*)), it is small areas of packed soil.

As an indication of the effect of slope on run-off from sodded areas such as lawns, a number of diagrams were drawn of the St. Louis sprinkling experiments of 1923 (filed with the record manuscript in Engineering Societies Library). Three runs each from Plot *B* (5% slope) and from Plot *C* (0.8% slope) are analyzed, using rainfall rates of $2\frac{1}{2}$ in. (adjusted), and comparing run-off rates that seem to prevail for some time (30 to 40 min) after run-off begins.

A rough comparison of values in Table 9, in the absence of pondage, indicates that the run-off from good slopes may be 15% greater than from practically flat areas. Measured run-off rates from Area *A* (3% slope) are nearly two-thirds greater than from Area *B* (0.7% slope), but the former is also less pervious, while pondage is prevalent in the latter. These data indicate a difference in the water lost—that is, in water not appearing as run-off—of 11% as between the 0.5% slope and the 5% slope.

TABLE 9.—Effect of Slope on Run-Off from Sodded Areas

(a) Plot *B*, 5% Slope						(b) Plot *C*, 0.8% Slope					
Date	Ground conditions	Run-Off Rate, in Units:		Loss		Date	Ground conditions	Run-Off Rate, in Units:		Loss	
		Each observation	Average	Rate, in units	Percentage			Each observation	Average	Rate, in units	Percentage
May 9, 1923	Damp	2.00	1.93	0.57	20	Aug. 31, 1923	Damp	1.90	1.73	0.77	31
July 26, 1923	Dry	2.00				May 12, 1923	Dry	1.70			
Aug. 1, 1923	Wet	1.80				July 26, 1923	Damp	1.60			

Within narrow limits, these values may be used to adjust, for slope, the coefficients in Table 8; for example, if the city block had all the characteristics of Area *A*, except that the slope is 2% instead of 3%, the coefficient of the pervious portion could be reduced from 0.45 to about 0.39. This should probably be charged against a slightly higher rate than the nominal run-off, to allow for water spilling from yard walks on to lawns, a condition equivalent to an increased rainfall rate on the lawns. In this instance, the revised run-off might be taken as 50% of 39% of 3 in. as compared with 50% of 45% of 3 in., a reduction in average run-off rate for the block from 1.89 to 1.80. Of course, modifications of this kind cannot be used too extensively or too far from the field of original figures. If used for very flat areas, such as Area *B*, allowance should be made for the effect of increased pondage.

The effect of soil characteristics on run-off rates, and on the probable frequency of occurrences of specific run-off rates, limits the direct value of the presented data to St. Louis conditions, or where all conditions are sub-

stantially equivalent to those of one of the blocks studied. The results can be modified. however, to be of value for other localities.

The volumetric ratio has proved to be the best index to the varying value of the ratio between run-off rates and rainfall rates during the progress of a storm. It provides a convenient means of comparison between different types of inlet areas (see percentile curves, Fig. 18(*a*)). It has given remarkably uniform and consistent results when applied to the sprinkling experiments of St. Louis and Dallas, Tex. (Complete supporting data are filed with the record manuscript, in Engineering Societies Library.) This ratio is suggested for use in adjusting run-off rates to other soil types, using the Dallas information, which is given in part in Fig. 28 and in Tables 10 and 11, as an example.

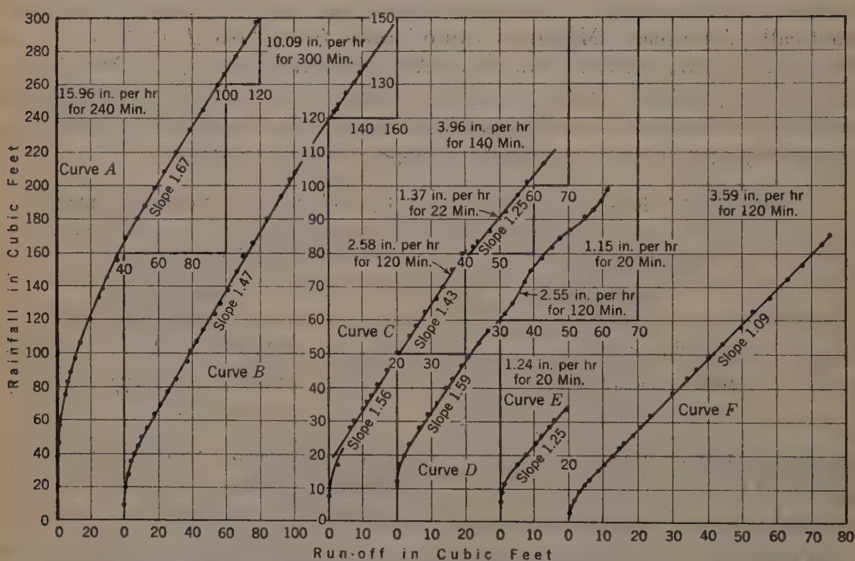


FIG. 28.—VOLUMETRIC RATIOS OF RAINFALL TO RUN-OFF, 1931–1932; SPRINKLING EXPERIMENTS, CITY PARK, DALLAS, TEX. (SEE TABLE 11).

TABLE 10.—VOLUMETRIC RATIOS OF RUN-OFF TO RAINFALL; SPRINKLING EXPERIMENTS (NO PONDAGE)

Item No.	(*a*) EXPERIMENTS AT ST. LOUIS, MO.					(*b*) EXPERIMENTS AT DALLAS, TEX.				
	Plot	Year	Soil condition	Ratio		Plot*	Season	Soil condition	Ratio	
				Range	Average				Range	Average
1...	*A*	1922	Bare	0.88–0.98	0.93	*A*	Spring	Sodded	0.62–0.80	0.74
2...	*A*	1923	Sodded	0.54–0.66	0.61	*A*	Fall	Sodded	0.52–0.66	0.60
3...	*B*	1922	Bare	0.70–0.95	0.86	*B*	Spring	Sodded	0.63–0.80	0.72
4...	*B*	1923	Sodded	0.60–0.86	0.76	*B*	Fall	Sodded	0.54–0.60	0.57
5...	*C*	1922	Bare	0.50–0.74	0.63	*C*	Spring	Sodded	0.82–0.95	0.89
6...	*C*	1923	Sodded	0.66–0.76	0.71	*C*	Fall	Sodded	0.52–0.63	0.58

* Plot *A* is Southern Methodist Univ.: Plot *B* = City Park Plot; and, Plot *C* = Exall Park Plot.

Plot *B*, Fig. 29, is comparable with the City Park Plot in Dallas, in that the slopes are about the same and both are sodded; and a direct comparison

TABLE 11.—OBSERVATIONS TO DETERMINE VOLUMETRIC RATIOS OF RAINFALL TO RUN-OFF; SPRINKLING EXPERIMENTS, CITY PARK, DALLAS, TEXAS (SEE FIG. 28).

Curve	Date	Time	Remarks
A	November 7, 1931....	9:30 A.M...	Dry; heavy dew on a good stand of thick, heavy, green grass.
B	November 9, 1931....	9:00 A.M...	Fair and cool; light wind; damp.
C	November 10, 1931....	8:30 A.M...	Cloudy; wet.
D	March 1, 1932........	12:30 P.M...	Cloudy; warm.
E	March 2, 1932........	....	Cloudy; cool; ground soft and damp; heavy dew.
F	March 5, 1923........	....	Ground saturated and frozen; fair, cold, and windy.

of the soil conditions on the two plots may be made by comparing their volumetric ratios between run-off and rainfall. In Table 10 it is seen that this ratio for Plot *A* in St. Louis (1923) is 0.76. For the City Park Plot in Dallas the ratio for spring rains is 0.72 while for fall rains it is 0.57.

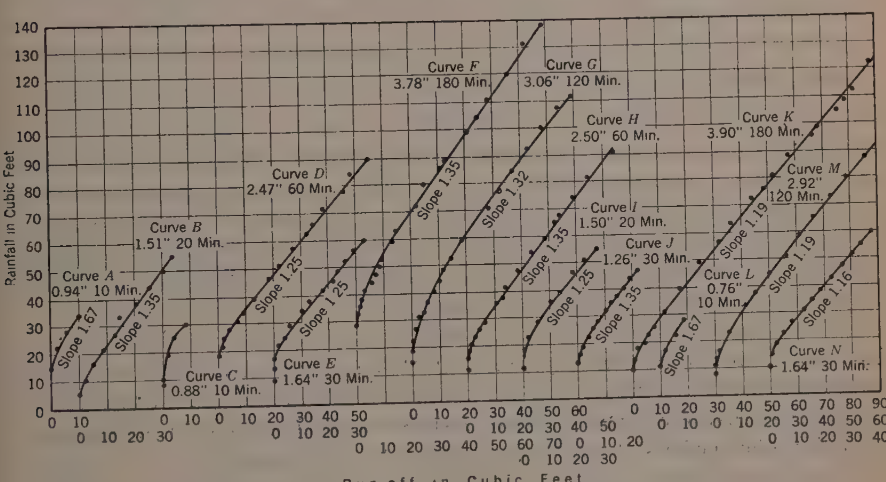


FIG. 29.—VOLUMETRIC RATIOS OF RAINFALL TO RUN-OFF, 1923; SPRINKLING EXPERIMENTS, PLOT *B* (SODDED), ST. LOUIS, MO. (SLOPE, 5.1 PER CENT).

Thus, the first important difference in conditions at the two localities becomes apparent; the correlation diagram, Fig. 30, shows that there is no marked difference in this ratio between spring and fall rains at St. Louis.

It is evident that if the Dallas plot is representative of general soil conditions in that city, the St. Louis run-off curves, Fig. 23 (ignoring frequencies, for Station *A* (Belt and Ridge Avenues), for example), could be modified to find the probable run-off from similar blocks in Dallas for given intensities and duration periods. For winter and spring rains at Dallas, little or no adjustment of St. Louis ratios would be necessary since the corresponding ratios are practically equal. For summer and fall rains, rates derived from the St. Louis data would need to be adjusted in accordance with the relative losses shown in the sprinkling experiments. For St. Louis, the water losses through surface film and absorption are given

as $\frac{100 - 76}{100} = 24$ per cent. For Dallas, the corresponding loss is $\frac{100 - 57}{100}$ $= 43$ per cent. The difference in water loss, therefore, is 19% of the rain-

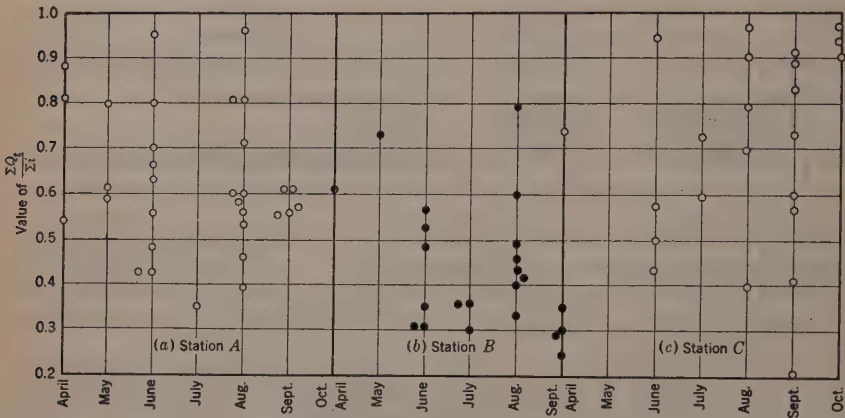


FIG 30.—CORRELATION DIAGRAMS; SEASONAL VARIATION OF VALUE $\frac{\Sigma Q_t}{\Sigma i}$, ST. LOUIS EXPERIMENTS

fall. If all other conditions are equal, except that it is desired to alter Item No. 4, Table 8, for summer and fall soil conditions at Dallas, the computation for the pervious portion (Table 8, Item No. 4, Column (4)) would involve the reduction of the factor 45% by 19, or to 26%, and this, again, should probably be chargeable against a Dallas intensity of a frequency equivalent of the St. Louis rainfall rate of 3 in. per hr.

In this particular case, it would probably be advisable to draw two independent rainfall frequency curves for Dallas, one for winter and spring rains and one for summer and fall rains. For a given design frequency, it might be that for some duration periods the spring curve would govern, while for other periods the fall curves might be indicated.

CONCLUSIONS TO PART IV

In general, it is suggested that the data presented may be made useful in localities other than St. Louis, by well-considered modifications in four respects:

(*a*) An adjustment of the run-off curves of Fig. 23 in the direction and proportion that rainfall curves of the new locality may bear to the 61-yr rainfall curve given in Fig. 23(*b*).

(*b*) With regard to varying percentages of impervious area, as suggested in Table 8.

(*c*) For slight differences of slope as outlined in Table 7.

(*d*) For other soil types as in the last paragraph.

Somewhat better information would be made available through such modifications than has generally been the case. It is highly desirable that additional tests and repetitions of these experiments be conducted in other localities in the near future.

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PAPERS

THE SILT PROBLEM

By J. C. Stevens,[1] M. Am. Soc. C. E.

Synopsis

All the basic data that the writer could secure on the silting of reservoirs, where actual capacity surveys have been made to determine the extent of silting, are contained in this paper. Remedial measures for silt elimination are presented and discussed. A table contains a brief of all data on the silt transported by the streams of the world. The physical laws of silt transportation are outlined, with pertinent discussion. The control of silt in canals, reservoirs, and on water-sheds is then considered. The paper closes with data and discussion on the origin of silt.

The original paper, containing additional data and more extended discussion, is on file in Engineering Societies Library, 29 West 39th Street, New York, N. Y.

Statement of the Problem

The word, "silt", as used in this paper, includes in its definition all material transported by flowing water whether carried in suspension or transported as bottom load. Silt originates from the disintegration of rocks by the climatic agencies of rain, wind, and frost, and by chemical agencies in water and air. The effect of such disintegration is evidenced by the wearing down of mountains, the gullying of their slopes, the filling of valleys, the extension of deltas into seas—in brief, the leveling of mountain ranges into plains.

Geological history is marked by a succession of sedimentations and uplifts, in which varying climates, water supplies, vegetation, glaciation, and volcanism have played and are playing all-important parts. The processes of disintegration, erosion, transportation, sedimentation, mountain building, and plain leveling are still in progress and are as effectual as ever.

Note.—Early in its existence the Special Committee on Irrigation Hydraulics selected the subject of "The Silt Problem" as one of ten for study and research. This paper was submitted to the Committee by its author, and the Committee has recommended its publication in *Proceedings*, in order to elicit discussion of the subject (see Progress Report of the Committee, *Proceedings*, Am. Soc. C. E., March, 1928, Society Affairs, p. 173).

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Existing topography is a complex residual of these processes in which unnumbered eons have contributed to its present configuration.

Man can not hope to halt the processes of mountain erosion and plain building. The land he cultivates could not exist except for these forces. He must expect that rains will gully his fields, or cover them with mountain débris, and that the streams will continue to carry sediments that will fill the canals and reservoirs.

Man's problem lies in utilizing the agencies of sedimentation to his advantage where possible and in opposing the nullification of his endeavors by controlling them through whatever forces lie at his command.

Data on Silting of Reservoirs

The construction of large storage reservoirs on some of the silt-laden streams of the West has focused attention on the part sedimentation will play in the future history of Western civilization.

An empire exists below the storage reservoir that has been created by the Elephant Butte Dam, on the Rio Grande, in New Mexico. The land is phenomenally productive. This region embraces a substantial unit of civilization the very existence of which hangs upon the integrity of a storage reservoir to impound and deliver the water so vitally necessary to life.

The reservoir created by Elephant Butte Dam is slowly being deprived of its ability to store water. Silt is being deposited at an average rate of 20 000 acre-ft per yr. Its original capacity will be so depleted in two or three generations that the civilization now dependent upon it will have to seek other sources of water supply and storage.

Fortunately, sites are available where other reservoirs may be constructed; and, after these are gone, others will doubtless be found. Furthermore, the present dam could be raised to increase its storage capacity; but what of the ultimate future, when all available storage sites have been exhausted? Must those now fertile areas revert, ultimately, to the sage-brush and the cactus? Will sedimentation, that made possible this vibrant civilization, ultimately sound its death knell? How long can this episode in the annals of civilization continue and what can Man do to prolong its existence?

The Boulder Dam, in the Colorado River, will create the largest storage reservoir in the world. Upon it will depend not only the security of the entire Imperial Valley against damages from floods, and the irrigation of millions of acres of land, but also the domestic and irrigation supply for the metropolitan district of Southern California. The ability to furnish power to the people in seven States also hinges upon its functioning. Unless remedial measures are adopted, this reservoir will become virtually useless, by reason of silt deposits, before the passing of the fifth generation.

Fortunately, as on the Rio Grande, a number of other storage sites are available on the Colorado River. Ultimately, however, within some definite number of generations, the fact must be faced that all these reservoirs will

TABLE 1.—RESERVOIRS FOR WHICH AMOUNT OF SILTING HAS BEEN MEASURED
(Computations Carried to Three Significant Figures)

Reservoir	Stream	Drainage area, in square miles	Mean annual supply, in thousands of acre-feet	Original Capacity*		Year of Capacity Survey*		Period, in years	Water supply during the period, in thousands of acre-feet	Silt Deposited				Reference No.†
				In thousands of acre-feet	Percentage of annual supply	First	Last			Total, in acre-feet	Annually, in acre-feet	Percentage of original capacity	Per thousand of water supply, by volume	
(a) Within the United States														
Boulder	Colorado, River, Ariz.	140 000	15 000	30 500	203	....	....	....	....	....	.....	......	......	(58), (6a)
Elephant Butte	Rio Grande, N. Mex.	30 000	1 200	2 640	224	1916	1925	8.67	10 800	178 000	20 500	6.7	16.3	(40a),
Roosevelt	Salt, Ariz.	5 670	840	1 370	164	1905	1925	20.0	....	101 000	5 050	7.4	......	(40b), (58)
Keokuk	Mississippi	119 000	46 000	370	0.80	1891	1928	15	640 000	112 000	7 460	30.2	0.18	(44)
Hales Bar	Tennessee, Tenn.	21 800	27 000	156.0	0.57	1913	1930	16.92	460 000	45 500	2 690	29.1	0.10	(70a)
Parksville	Ocoee River, Tenn.	600	950	97.0	10.2	1912	1930	18.75	17 100	20 800	1 110	21.5	1 21	(70b)
McMillan	Pecos, N. Mex.	......	300	90.0	30.0	1894	1932	39	12 000	50 000	1 280	55.5	4.20	(40c), (58)
Guernsey	North Platte, Wyo.	16 200	1 650	72.0	4.4	1927	1933	5.9	9 870	8 400	1 430	11 0	0.85	(52), (58)
Old Lake Austin	Colorado of Texas	38 200	2 000	49.3	2.7	1893	1900	6.75	16 000	23 600	3 500	48.0	1.50	(40d), (46)
Lake Worth	W. Fk., Trinity, Tex.	1 870	212	47.2	22.2	1915	1928	13.0	....	13 900	1 060	29.6	......	(40e), (46)
Cucharas	Cucharas, Colo.	......	......	43.0	......	1910	1932	22	....	7 850	357	18.3	......	(58)
Cheoah	Little Tennessee, Tenn.	1 620	3 030	41.6	1.38	1918	1930	11.83	35 900	4 350	367	10.4	0.12	(70c)
Sweetwater	Sweetwater, Calif.	181	17.2	36.3	205	1886	1927	39	740	6 170	158	17.0	0.84	(41), (58)
New Lake Austin	Colorado of Texas	38 200	1 910	32.0	1.7	1913	1926	13	25 600	30 600	4 900	95.6	1.20	(40d), (46)
Lake Chabot	San Leander, Calif.	42	21.7	17.0	78.0	1875	1923	48	854	3 700	77	21.7	4.30	(41), (58)
White Rock	White Rock Creek, Tex.	114	......	16.9	......	1923	1928	5	....	680	136	4.0	......	(40f)
Boysen	Bighorn, Wyo.	7 740	1 500	16.0	1.06	1911	1924	13	....	13 000	1 000	80.0	......	(46), (58)
Zuni	Zuni, N. Mex.	650	17.4	14.8	85.0	1906	1932	26.2	466	11 300	432	76.5	24.3	(40g), (58)
Gibralter	Santa Ynez, Calif.	220	18.6	14.6	78.0	1920	1931	11	206	2 100	190	14.4	10.0	(41), (58)
Lake Michie	Flat, N. C.	170	90.0	14.3	......	1906	1930	3	114	90	30	0.63	0.78	(11), (58)
Sterling Pool	Rock River, Ill.	8 760	3 500	13.7	0.39	1912	1930	18	....	2 010	111	14.7	......	(54)
Coon Rapids Pond	Mississippi, Minn.	19 000	4 400	8.0	0.20	1899	1931	16	....	1 380	86	17.0	......	(69)
Furnish	Umatilla, Ore.	1 200	485	5.50	1.16	1909	1931	22	10 200	4 500	204	82.0	0.44	(58)
Lake Penick	Clear Fk., Brazos, Tex.	2 250	111	3.09	2.70	1920	1927	7	....	965	138	32.2	......	(40h), (46)
LaGrange	Tuolumne, Calif.	1 500	1 970	2.33	0.12	1895	1931	36	70 000	1 940	54	83.0	0.028	(7a), (58)
Buckhorn	Buckhorn, Colo.	130	......	1.19	......	1907	1925	18	....	565	31.4	47.5	......	(40b), (58)
(b) Outside the United States														
Aswan	Nile, Egypt	620 000	66 000	4 400	5.7	1900	1927	25	....	0	0	0	0	(9),(24);(36a)
Burrinjuck	Murrumbidgee, Austral.	5 000	1 150	772	67.0	1910	1924	14	14 700	3 970	285	0.51	0.28	(58)
Dhukwan	Betwa, India	8 240	......	85.4	......	1910	1932	.....	....	....	.....	25.0	......	(58)
Pericha	Betwa, India	10 300	......	67.5	......	1885	1932	.....	....	....	.....	0	......	(58)
Habra	Habra, Algeria	......	......	24.3	......	1885	1907	22	....	14 200	650	58.0	......	(45), (5a)
Helena	Mundaring, W. Austral.	......	48	17.2	36.0	1900	1930	30	1 500	690	23	4.0	0.46	(58)
Hamiz	Hamiz, Algeria	......	6.0	11.4	......	1872	1901	29	....	1300	45	11.4	......	(45), (5b)

* To spillway level or to a level controlled by flash-boards or crest-gates. † See corresponding numbers in Appendix.

have become useless for storage purposes. The dependent peoples must then be reduced to those that can subsist on the areas which the unconserved flow of the river will irrigate.

It is not the writer's intention to paint a dark picture, but rather to stimulate a more intensive study and an intelligent research that will ultimately effect a practical solution of this problem. The menace exists; it is real; and unless something constructive can be evolved, civilization in these regions must eventually decline. It is unfair to sit smugly complacent and to pass this problem flippantly on to future generations. The engineer should be equal to the task of finding a solution, but it will take many years of experimentation and study, and he should be at the task, amply financed.

The results of the sedimentation of reservoirs for which the extent of depositions have been determined by actual surveys, are given in Table 1. In order to make the data comparable, capacities and silt deposits have been referred to the controlled spillway level. On some reservoirs the silts deposited above this level may be a considerable part of the total, but this does not rob the reservoir of storage capacity. The following supplementary data pertaining to Table 1 are offered.

Boulder Reservoir (Under Construction).—Of the total capacity of this reservoir, 9 500 000 acre-ft. will be reserved for flood protection, 7 000 000 for silt, and 14 000 000 for irrigation. The maximum recorded discharge was 210 000 cu ft per sec on June 18 and 19, 1921. The maximum known flood occurred in 1884, for which the discharge was between 250 000 and 300 000 cu ft per sec (6*a*).[2] The spillway capacity is 400 000 cu ft per sec, exclusive of outlets for power and irrigation.

Elephant Butte Reservoir.—Capacity surveys for silt deposits in this reservoir were made in 1916, 1920, and 1925, with the results shown in Table 2(*a*). The maximum recorded mean daily discharge entering the reservoir area was 33 000 cu ft per sec on October 11, 1904. The overflow spillway capacity is 16 000 cu ft per sec. In addition, there are four regulating gates with a combined capacity of as much more.

Roosevelt Reservoir at Junction of Salt River and Tonto Creek.—This reservoir is 70 miles northeast of Phoenix, Ariz. Capacity surveys for silt content have been made as shown in Table 2(*b*). The maximum recorded inflow was 94 800 cu ft per sec on January 19, 1916. The spillway capacity is 150 000 cu ft per sec.

Keokuk Reservoir, at Keokuk, Iowa.—The Keokuk Reservoir was completed in 1913. It extends 42 miles up stream from the dam, and has a surface area of 25 200 acres and a capacity, at Elevation 518, of 370 300 acre-ft based on a survey made in 1891. The maximum recorded inflow was 314 000 cu ft per sec in May, 1888. Since the dam was constructed, the maximum flood was 242 000 cu ft per sec on April 16, 1922.

Hales Bar Reservoir.—The dam is near Guild, Tenn., and was completed November 1, 1913. The normal pool level (crest of dam) is at Elevation 626.2. The original capacity of the reservoir to the crest of the

[2] Figures in parentheses refer to numbered articles in the Bibliography (Appendix).

TABLE 2.—CAPACITY SURVEYS FOR SILT DEPOSITS
(Computations Carried to Three Significant Figures)

Period of record	Years	Silt Deposited: In acre-feet	Silt Deposited: In acre-feet per year	Silt Deposited: Cumulative total, in acre-feet	Reservoir Capacity: In acre-feet	Reservoir Capacity: Percentage lost
(a) ELEPHANT BUTTE RESERVOIR						
1916	....	....	....	....	2 640 000	0
1916–1920	4.06	90 900	22 400	90 900	2 550 000	3.3
1920–1925	4.61	86 900	19 000	178 000	2 460 000	6.7
(b) ROOSEVELT RESERVOIR						
1905	....	....	....	....	1 370 000	0
1905–1914	9.0	27 000	3 000	27 000	1 340 000	2.2
1914–1916	2.0	35 000	17 500	62 000	1 310 000	4.4
1916–1919	3.0	0	0	62 000	1 310 000	4.4
1919–1925	6.0	39 000	6 500	101 000	1 270 000	7.4
(c) PARKSVILLE RESERVOIR (70d)						
1912	0	....	....	....	97 000	0
1912–1921	9	15 600	1 720	....	81 400	16.1
1921–1930	9.75	20 800	1 110	....	76 200	21.5
(d) McMILLAN RESERVOIR						
January, 1894	0	0	0	0	90 000	0
1894–1904	10.42	16 000	1 540	16 000	74 000	17.7
1904–1910	6.42	12 000	1 870	28 000	62 000	31.1
1910–1915	4.58	17 000	3 710	45 000	45 000	50.0
1915–1932	17.58	5 000	285	50 000	40 000	55.5
(e) AUSTIN RESERVOIR						
1913	....	....	....	....	32 000	0
1913–1922	9	26 700	2 970	21 700	5 300	83.5
1922–1924	2	2 460	1 230	29 200	2 800	91.2
1924–1926	2	1 420	710	30 600	1 400	95.6
(f) SWEETWATER RESERVOIR						
1895	....	....	....	....	36 300	0
1888–1895	7	1 200	172	1 200	35 100	3.4
1895–1916	21	4 190	220	5 390	30 900	4.9
1916	11	773	70	6 160	30 100	1.70
(g) LA GRANGE RESERVOIR						
1895	....	....	....	....	2 330	0
1895–1905	10	1 260	126	1 260	1 070	55.0
1905–1911	6	430	72	1 690	640	72.6
1911–1931	10	250	25	1 940	390	83.2

dam was 142 000 acre-ft; 3 ft of flash-boards added later, increased this capacity to 156 000 acre-ft. Surveys were made of the Tennessee River in 1889–1891 by the United States Corps of Engineers. In 1905, the Power Company made a survey for flowage areas. In the fall of 1930, the U. S. Engineers made a capacity survey of the entire reservoir.

Parksville Reservoir (Also Called Ocoee No. 1).—The dam is 12 miles above the mouth of the Ocoee River, at Parksville, Tenn. About 70% of the drainage area is forest and wood lots. The Ducktown Mining District presents a striking example of the destruction of forest lands by the fumes from smelters, roast ovens, and incidental activities. About 20 sq miles have been completely denuded. Erosion is excessive as the poisonous gases

will not permit the growth of any vegetable matter. This condition, and the clayey soils and excessive rainfall to which this basin is subject, offers a complete explanation of this most flagrant example of unnatural erosion.

Surveys to determine the silt content were made by The Tennessee Electric Power Company in 1917, 1921, and 1929, and by the U. S. Engineers in the fall of 1930. The early surveys included only the up-stream half of the reservoir, but that of 1930 covered the entire reservoir. The results of the capacity surveys (70*d*) are given in Table 2(*c*).

Lake McMillan North of Carlsbad, N. Mex.—The original dam was completed by the Pecos Irrigation District, in 1894, with a capacity of 32 500 acre-ft at the spillway (Elevation 3 258.9). The project was greatly damaged by floods in 1906 and thereupon was taken over by the United States Bureau of Reclamation. The spillway has been raised repeatedly, the last time in 1915 to Elevation 3 267.7, to which the capacity surveys in Table 2(*d*) have been referred. The great reduction in the rate of silting since 1915 has been due to a dense growth of tamarisk (salt cedar) on the flats at the upper end of the reservoir. This growth induces the river to drop most of its silt in the valley just above the reservoir. Fig. 1 shows

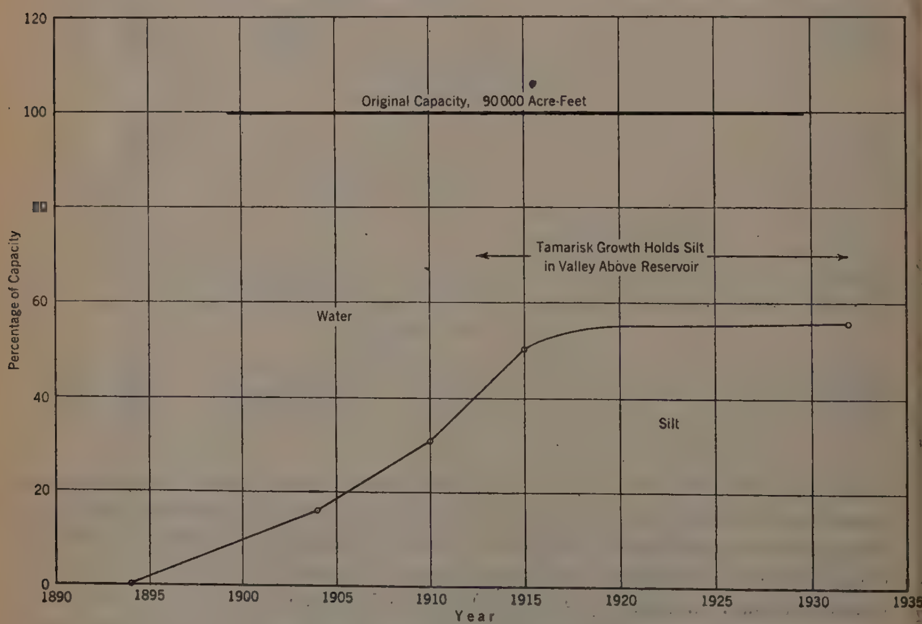

FIG. 1.—SILTING OF LAKE MCMILLAN.

the silting curve for this reservoir. Nearly one-third of the silt deposited during the last period resulted from the flood of April, 1915, during which the maximum recorded inflow of 42 000 cu ft per sec was observed. The nominal spillway capacity at the dam is 35 000 cu ft per sec.

Guernsey Reservoir.—This is a regulating reservoir on the North Platte River near Guernsey, Wyo. Storage began in March, 1927. Surveys for silt were made by soundings in February, 1929, showing 3 010 acre-ft of deposits; in January, 1931, 5 970 acre-ft; and in February, 1933, 8 400 acre-ft. Silt was found mostly in the old river channel in the up-stream half of the reservoir. The normal flow of 1 650 000 acre-ft is the average inflow for the period, 1909–1932. The maximum discharge was 21 000 cu ft per sec, on June 28, 1917.

Austin Reservoir, Austin, Tex.—The first dam was completed in May, 1893, and failed in April, 1900. In was rebuilt in 1911–1913. During the interval, 1900 to 1913, much of the silt deposited in the old reservoir was washed out. The new reservoir, with a spillway 9 ft lower than the old, had 65% of the capacity of the old reservoir when it was first filled in 1913. Capacity surveys were made in the late summer of the years indicated in Table 2(*e*). They refer to the new reservoir.

The spring flood of 1922 deposited more silt in two weeks than had accumulated in the previous five years. The maximum recorded discharge over the spillway was 151 000 cu ft per sec on April 7, 1900, just before the dam failed.

Lake Worth, Near Fort Worth, Tex.—The average water supply from October 16, 1923, to September 30, 1930, was 211 800 acre-ft per yr. This reservoir provides water for the City of Fort Worth. The maximum recorded discharge was 7 600 cu ft per sec, on November 18, 1923.

Cucharas Reservoir, Arkansas River Drainage East of Pueblo, Colo.—The drainage area is 6 000 to 12 000 ft high, composed of narrow valleys and eroded mesas sparsely timbered, except in the higher altitudes. No actual surveys of silt content have been made, but it is known to be silted to an average elevation of 68.0 ft (spillway level, 106.0 ft), for which the capacity is 7 850 acre-ft.

Cheoah Reservoir at Mouth of Cheoah River, Near Fairfax, Tenn.—The total drainage area of the Little Tennessee River is 2 650 sq miles, of which 60% is forest lands, most of which have been cut over. The Cheoah Dam and Hydro-Electric Plant were completed in 1919. The original capacity at normal pool level (Elevation 1 275.8) was 41 600 acre-ft. The Aluminum Company of America made a survey for silt content in the upper 2½ miles of the reservoir in 1922. The U. S. Engineers made a complete survey of the reservoir in the fall of 1930.

Sweetwater Reservoir.—This reservoir is in San Diego County, California. It was originally built in 1888, to a height of 70 ft above the outlet. The spillway was raised 5 ft in 1896. The height of the dam and spillway was again increased 15 ft in 1915. The present (1932) spillway is 90 ft above the outlet; capacity surveys were made in 1887, 1895, and 1917, and a partial survey in 1927, with the results shown in Table 2(*f*). Most of the silt was deposited in flood years. Stream-flow records cover 42 climatic years, from 1887 to 1929. The maximum inflow occurred in 1915–16 (160 000 acre-ft). The minimum was zero during the four years ending

September 30, 1900, 1902, 1903, and 1904. During nine years (1895–1904), the total water supplied was only 9 290 acre-ft. The maximum recorded discharge was 45 500 cu ft per sec, on January 27, 1916.

Lake Chabot, Southwest of Oakland, Calif.—The drainage area is densely covered with brush and redwood. Capacity surveys referred to Spillway Gauge 83.5 are: For 1875, 17 000 acre-ft; for 1900, 15 500 acre-ft; for 1911, 13 800 acre-ft; and for 1923, 13 500 acre-ft.

White Rock Reservoir.—This reservoir is 4 miles east of Dallas, Tex. Of the drainage area, 75% is cultivated land. A survey in 1910 (not very accurate) gave a capacity of 21 500 acre-ft at Elevation 140.15 (local datum). The survey of 1923, which was made accurately, showed a capacity at Elevation 140.5 of 19 535 acre-ft, and 16 896 acre-ft, at Elevation 138.5 (spillway level).

Boysen Reseroir in Fremont County, Wyoming.—The dam was completed in 1911 to create head for a hydro-electric plant. Storage was of secondary consideration. Soundings were made through the ice on December 5, 1922, and again on January 23, 1924. On the latter date the reservoir was practically full of silt to the spillway level. The deposit of 13 000 acre-ft of silt is largely estimated. The power plant has carried no load since May, 1928, on account of silt accumulations.

Zuñi Reservoir.—The reservoir is at Blackrock, N. Mex., in the Zuñi Indian Reservation. The dam was completed in 1907. Capacity surveys have been made by sounding through the ice during each year when it was possible. When the ice did not cover the entire silt deposits, the reservoir topography from the preceding survey was used for that part above the ice. The deposits between ice level and top of flash-board level (Elevation 998.3) for some of the years are, therefore, probably not included in the totals given in Table 3.

TABLE 3.—CAPACITY SURVEY OF ZUÑI RESERVOIR IN NEW MEXICO

Date of survey	Period, in years	Capacity, at Elevation 998.3, in acre-feet	Run-off for period, in acre-feet	Silt Deposited During Period		Capacity Lost	
				In acre-feet	Per thousand of inflow	Cumulative, in acre-feet	Percentage of original
1906...	....	14 800	......	......	......	......	......
June 26, 1910...	4.0	13 000	56 200	1 800	32	1 800	12.2
December 22, 1911...	1.5	11 800	24 600	1 200	48	3 000	20.2
January 1914...	2.1	10 600	14 500	1 200	83	4 200	21.4
January 1918...	4.0	9 240	141 000	1 360	9.6	5 560	37.6
January 1919...	1.0	8 560	5 430	680	125	6 240	42.1
January 1920...	1.0	7 310	46 400	1 250	27	7 490	50.6
January 1921...	1.0	6 500	9 640	810	84	8 300	56.0
February 10, 1922...	1.1	4 880	7 000	1 620	231	9 920	67.0
January 1924...	1.9	4 500	22 900	380	17	10 300	69.5
January 1925...	1.0	4 390	8 880	190	21	10 490	71.0
January 1926...	1.0	4 120	5 170	170	33	10 660	72.0
January 1927...	1.0	3 950	4 980	170	34	10 830	73.0
December 24, 1927...	0.9	3 500	16 000	450	28	11 280	76.0
December 29, 1928...	1.0	3 620	55 600	—120	Scour	11 160	75.0
January 29, 1930...	1.1	2 820	15 100	800	53	11 960	80.6
July 1932...	2.6	3 450	33 200	—630	Scour	11 330	76.5
Total........	26.2	....	466 000	11 330	24.3	....	....

The survey of July, 1932, was a topographic survey of the entire reservoir taken on the surface of the silt deposits while the reservoir was practically empty. It is probably the most accurate survey that has been made.

Protective works to hold the silt on the water-shed were begun in 1923 on Rio de Los Nutrias, the principal silt-producing tributary. In July, 1931, a hole was blasted in the gate tower for the installation of a 4 by 6-ft sluice-gate, which was installed on October 15, 1931. Between these dates 500 acre-ft of silt were sluiced from the reservoir. Sluicing operations have continued each year whenever water was available. The gain in capacity for the surveys of 1928 and 1932 is partly due to sluicing and perhaps partly to inaccuracies in the preceding surveys. Fig. 2 shows the rate of silting. All capacities are referred to the top of the present flash-boards (Elevation 998.3, old datum, = 6 637.1, United States Geological Survey datum).

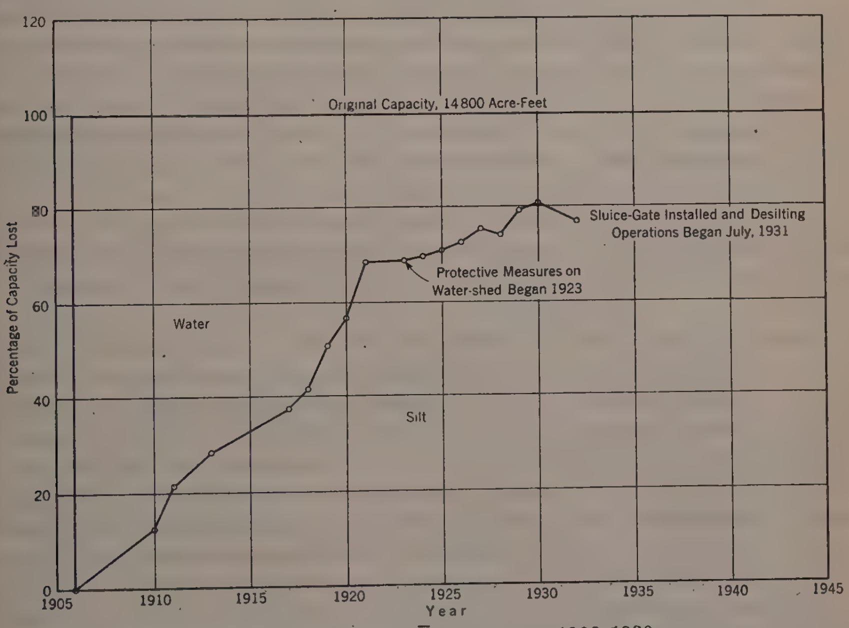

FIG. 2.—SILTING OF ZUÑI RESERVOIR, 1906–1930.

Gibraltar Reservoir.—Water for the City of Santa Barbara, Calif., is stored in this reservoir. The drainage area is sparsely covered with brush and small trees. Most of the silt came after the fire of 1923, which burned over the greater part of the water-shed. The water supply is very erratic. The year, 1921–22, yielded 65 500 acre-ft, whereas in 1923–24 the inflow was only 2 000 acre-ft. The maximum mean daily discharge (outflow) was 7 250 cu ft per sec, on April 8, 1926.

Lake Michie.—This reservoir stores water for the City of Durham, N. C. Ten permanently marked cross-sections of the reservoir were established just before filling it in the fall of 1926. Silting observations were made at

these cross-sections in September, 1930, showing an average silt deposit of 8 to 10 in. Some of this deposit was due to wave action washing the top soil of the lake margin into the lake. The normal flow of 124 cu ft per sec is the average of the years, 1925–26 to 1929–30, inclusive.

Sterling Pool.—This is formed by the Government dam at Sterling, Ill. The dam and locks were constructed in connection with a feeder for the Illinois and Mississippi Canal. Soundings were made at a series of cross-sections above the dam in 1912–13.[8] The soundings in 1930 were made from the same base lines. During the 18-yr period a total of 3 360 000 cu yd of silts were deposited, while 123 000 cu yd of material were scoured. The maximum recorded flood was that of May 16, 1929 (29 200 cu ft per sec, 24-hr mean).

Coon Rapids Pond, Anoka, Minn.—This pond is used solely for power purposes, and its level is kept constant; no storage is utilized. The plant was completed in August, 1914, at which time the pond was first filled. Surveys with soundings of the area to be occupied by this pond were made in 1874 by the U. S. Engineer Office at St. Paul, Minn.; in 1899, by the Mississippi River Commission; and, again, in January, 1931, by the U. S. Engineers at St. Paul. Evidences of general scouring were detected between 1874 and 1899. General silting occurred between 1899 and 1931, which is assumed to have occurred since 1914.

There are dams at St. Cloud, Minn. (60 miles above Coon Rapids) on the Elk River, and on the Run River. These three control about 15 000 sq miles of the drainage area, but, obviously, they do not intercept all the silt.

Furnish Reservoir, Near Pendleton, Ore.—By 1914, this reservoir had silted so badly that it became necessary to put flash-boards on the spillway in order to store sufficient water for the lands under the Furnish Ditch. The use of flash-boards was discontinued in 1930. The sluice-gate in the dam is opened as soon as the storage has been exhausted, permitting the river to flow through the Furnish Reservoir. This cuts a channel through the silt to the original gravel bed. Thus, a certain amount of desilting is done each year. The survey of 1930 is little more than an estimate by the Water Master. In 1932, the writer estimated the remaining storage capacity at about 600 acre-ft.

Lake Penick.—This reservoir stores water for Stamford, Tex. Of the drainage area, 30% is farming and 70% grazing land. The dam was begun in 1918. The run-off at the U. S. Geological Survey gauging station at Nugent, with a drainage area of 2 220 sq. miles (1924 to 1930), showed 680 200 acre-ft, or 111 000 acre-ft per yr. The maximum discharge was 11 500 cu ft per sec on May 20, 1928.

LaGrange Reservoir.—The diversion dam is for the Turlock and Modesto Irrigation Districts, near LaGrange, Calif. Surveys for silt content have been made as shown in Table 2(*g*). The Don Pedro Dam completed in 1923 is 6 miles up stream. When its spillway was first used

[8] H. R. Doc. No. 964, 63d Cong., 2d Session.

much river débris was washed into the LaGrange Reservoir. The maximum recorded discharge was 38 100 cu ft per sec on March 25, 1928.

Buckhorn Reservoir Near Loveland, Colo.—No records of stream flow have been kept. The reservoir had a capacity of 1 191 acre-ft upon completion in 1907. In 1925, surveys were made for increasing its capacity when it was found that the storage capacity had been reduced to 626 acre-ft by silt. The maximum flood of 10 500 cu ft per sec occurred on June 15, 1923 (61*a*).

Aswan Dam and Reservoir, Egypt.—The first dam was built to store water to a reservoir level (R. L.) of 106.0 m, with a capacity of 865 000 acre-ft. It was completed in 1902. No spillway was provided, but the dam has 180 sluice-gates to pass a maximum flood of 500 000 cu ft per sec. The dam was raised beginning in 1907 and completed in December, 1912, for storage to R. L. 113.0 m, providing a capacity of 1 970 000 acre-ft. A contract was let in 1929 to raise the dam again, this time to store water to R. L. 122.0 m, for which the capacity will be 4 400 000 acre-ft. The average annual flow of the Nile at Aswan (1912 to 1927) was 90 000 cu ft per sec, the maximum being 500 000 cu ft per sec.

Burrinjuck Reservoir.—Water for Murrumbidgee irrigation areas, in New South Wales, is stored in this reservoir. Construction was begun in 1908, and was completed by 1920. The spillway crest is at R. L. 1 180. The silt deposits were deduced from regular samplings above slack water, 1910 to 1916, and above slack water and below the dam, 1917 to 1924.

Dhukwan Reservoir.—This reservoir is above the mouth of Jamni River near Jhansi, United Provinces, India. The drainage area is without snow storage, and the silt deposits were estimated.

Pericha Reservoir.—This is 30 miles below the mouth of Jamni River and 35 miles below Dhukwan Reservoir. Silt deposits are reported to be negligible. The reason for silting in Dhukwan Reservoir and not in this reservoir could not be obtained. Silt interception by Dhukwan Reservoir does not fully explain this phenomenon, since it is 23 years old and has lost one-fourth of its capacity, while Pericha Reservoir has lost none in 48 years.

Habra Reservoir in Algeria.—The dam broke in 1881 and all the silt was sluiced out. The reservoir was put in service again in 1885. During later years sluicing has been resorted to periodically. It is estimated that 4 900 acre-ft of silt have thus been sluiced out, so that the actual silt deposited should be increased by that amount. The river carries approximately 1% of silt by weight per annum.

Helena Reservoir.—The domestic water supply for Perth, Western Australia, is furnished by this reservoir. Surveys were made in 1913, 1920, and 1930. The stream flow into the reservoir from 1900 to 1930 registered a maximum per year of 151 000 acre-ft, and a minimum of 1 190 acre-ft.

Hamiz Reservoir, in Algeria.—Periodic sluicing was begun in 1901; 23% of the total water supply is used in sluicing out silt. No part of the silt deposited prior to 1901 has been removed but further loss of capacity has been prevented.

Sedimentation Processes

When a stream carrying a load of sediment into a reservoir meets the quiet waters of the reservoir its velocity is destroyed and its load of silt is deposited, forming a delta. In delta building the material is sorted as it is deposited. The finer material is carried far out into the reservoir and spread over the bottom. This part of the delta consists of the suspended load of the stream, sorted from coarse to fine and spread over a fan-shaped area the apex of which is at the mouth of the stream. For distinction this part of the delta is called the bottom set bed.

Superimposed on this is the stream's bed load deposited over a fan-shaped area, and called the foreset bed. These deposits are also sorted, the coarser material settling first. The foreset beds are deposited in layers or strata normally inclined to the horizontal at the slope of repose of the material in its saturated condition ((81) (82)). These strata vary in thickness and in inclination with every variation of the stream's bed load and rate of flow.

The process of building the foreset beds is somewhat analogous to that of building road grades by continually dumping material on the advancing face. If the advancing face were fan-shaped and each carload was of different material in size, hue, and condition of saturation, a faint picture of the process of laying down a typical foreset bed may be had.

The top surface of the typical foreset bed is fairly level provided the water surface of the reservoir remains constant. On top of this is deposited the suspended load of the stream in the same manner that the original bottom set beds were formed, except these deposits on top are of the coarser suspended material. This part of the delta is called the top set bed. The finer suspended material is carried beyond the foreset bed and deposited as bottom set bed in advance of the inclined strata of the foreset bed.

As the river flows over the top of the foreset bed, it may form ripples, dunes, and anti-dunes. Often the top set beds are superimposed over these dunes and ripples, preserving them intact.

If the lake level varies periodically, but with sufficient time for deltaic deposits to be made at each level, the delta is formed in benches. Of such would be those in certain natural lakes the levels of which vary only with cyclic weather changes, as, for example, Great Salt Lake. When the level varies seasonally, such as is the case with most artificial reservoirs that may be emptied and filled one or more times each year, the delta soon loses its typical characteristics.

Variability of stream regimen, variability of reservoir levels, and variability of materials transported, introduce great complexities. Deposits at one level are cut through at a lower level and re-deposited farther out; waves re-sort and flatten the slopes. The net result of these complex and interlocking processes may cause the deltaic shore to become a well-graded composite of coarse and fine material of considerably greater density than would be found in either of the delta beds in their idealistic state.

Where the reservoir capacity is small compared to the annual inflow, it may happen that a considerable part of the suspended load is carried through the reservoir and only the bed load is deposited. This is evidenced on many streams where the space above dams has been completely filled with coarse gravel and cobbles.

A sediment-carrying stream in a valley builds its bed and banks higher than the surrounding land. Alluvial valleys normally slope away from the stream channel. As the banks become higher the stream slope is reduced until finally a flood overtops and cuts through them and the river forms a new channel. The ends of the old channel soon become closed by silting and, thereafter, it stands as a lake until it is slowly filled by sediments from overflows and wind-borne materials.

A dam across a stream bed not only induces filling the slack-water space above it, but induces the deposition of river débris a long distance up stream. Fig. 3 is a profile of Bear Creek near Colfax, Calif., before and

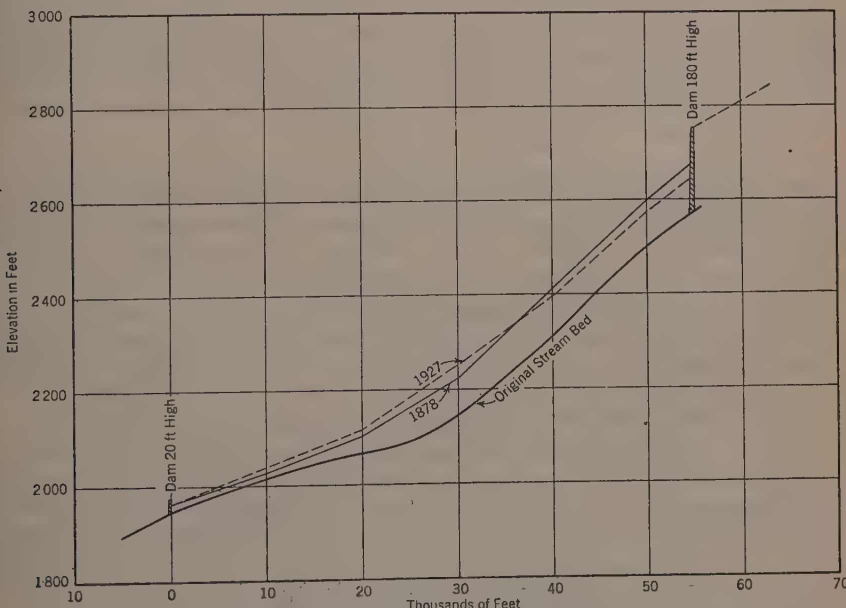

FIG. 3.—DÉBRIS BARRIER ON BEAR CREEK; CALIFORNIA DÉBRIS COMMISSION.

after the construction of a débris barrier. This comparatively low dam had the effect of withholding an enormous volume of coarse river débris from passing to the lower lands. This serves to illustrate an extreme case of aggrading a stream bed where only coarse sand, gravel, and boulders are concerned.

THE LIFE OF A RESERVOIR

A reservoir can only store water for subsequent use below its spillway level, or such other higher level as may be controlled by flash-boards or

crest gates. The silt deposited below spillway level, therefore, robs the reservoir of that much water-holding capacity. Ultimately, a reservoir on a silt-laden stream will fill up to the spillway level, leaving only a meandering stream channel on the alluvial plane made by these deposits.

Table 1 shows the average rate of silt deposition per annum. The useful life of a reservoir is not the number of years obtained by dividing its original capacity by the deposition rate per year. Such generalizations are not permissible. The usefulness of a storage reservoir may be practically exhausted when its capacity has been reduced 50%, or some other substantial part of its original capacity. On the other hand, a reservoir for power purposes may still be useful after it is completely filled.

The rate of silt deposition diminishes as the reservoir fills. Deltas will form at the mouths of all tributaries. As the deposits fill the storage space a continually lessening volume of quiet water is available for deposition of the finer materials, with the result that a continually increasing part of the material entering is carried over the spillway. During extreme floods earlier deposits may be picked up and carried out of the reservoir. Such effects are more apparent in reservoirs the capacities of which are a relatively small part of the annual flow through them.

Space Occupied by Silt

The relation between weight and volume of silt is one of almost infinite variation. There is no fixed relation. The specific gravity of all silts is quite constant. The difference in unit weight of silt, therefore, is almost entirely a matter of the voids.

Assuming 2.6 as a fair average of the specific gravity of the silt particles, the weight per cubic foot of dry and saturated silt may be determined as shown in Table 4.

TABLE 4.—Computation: Weight of Dry and Saturated Silt

Percentage of voids	Weight per Cubic Foot		Percentage of voids	Weight per Cubic Foot	
	Dry silt	Saturated silt		Dry silt	Saturated silt
0	163	163	50	81	113
20	131	144	60	65	102
30	114	133	67.5	53	95
40	98	123	70	49	93
47.8	85	115	80	33	83

When silt is first deposited it is loose and flocculent, and the finer the silt the greater the volume it occupies. As it lies in place and more sediment is superimposed, it becomes more and more compact. Some have contended that the water pressure tends to compact the silt. Borings in silt deposits in California reservoirs (41) have shown: (*a*) That a definite water-table exists in the deposits commensurate with the water level of the reservoir; (*b*) that the deposits below the water-table are flocculent; and

(*c*) that deposits do not shut off spring water inflow on the beds of reservoirs. Under such conditions the silt deposits are not consolidated by water pressure but by superimposed silt.

A most potent factor in compacting silt is exposure and consequent drying. The shrinkage of these deposits is evidenced by the cracks that form in a sun-dried exposure of fine silts.

Sands are not subject to such shrinkage. The amount of shrinkage, therefore, depends largely on the proportion of sand to clay and on the fineness of the clay. The more uniform the size of the particles the greater the voids and the less the specific weight. Deltaic deposits subject to wave action at all stages may become well graded from fine to coarse and, therefore, may be very dense.

These observations serve to explain why such disparity has been found in the volume-weight relation in various localities and by various investigators. Follett (48(*a*); 47) used a specific weight of 53 lb per cu ft for the Rio Grande silt. This was determined by selecting a single 3-in. cube from a sun-dried river bar. This sample was taken in 1904 and that weight has been used in all tables of the Rio Grande silt at San Marcial, N. Mex., since the beginning of the silt record in 1897. Humphreys and Abbot in 1861 used a weight of 120 lb per cu ft for the Mississippi River sediment.

Samples from exposed silt bars in various reservoirs in Texas showed the following (53):

Material	Dry weight, in pounds per cubic foot
Coarse silts at head of reservoir, probably consisted largely of bed load	92
Fine silts from much the same location	85
Deposits on surface of silt beds near middle of reservoirs	55
Finest material from submerged deposits in old river channels of reservoirs	31

Seventeen samples from exposed silt beds of Elephant Butte Reservoir in 1916 yielded the data presented in Table 5.

TABLE 5.—Samples from Exposed Silt Beds of Elephant Butte Reservoir

Description	Maximum	Minimum	Average
Weight as taken, in pounds per cubic foot	124.3	96.7	104.7
Moisture, percentage	20.9	4.4	11.6
Weight dry, in pounds per cubic foot	101.2	87.9	92.3
Specific gravity	2.66	2.59	2.64
Percentage of voids	46.6	39.0	44.0

In India (20*a*) engineers have found that the silts of the Indus River, when wet, averaged 95 lb per cu ft and that they contained 45 lb of water and 50 lb of dry matter. Many determinations have been made of the

specific weight of the silts of the Colorado River with greatly varying results (42*a*). A summary of these determinations is, as follows:

Location	Number of samples, mean of:	Dry weight, in pounds per cubic foot
Yuma, Ariz.	20	85.4
Laguna Dam	10	81.6
Bed silts: Imperial Canals, 1925	17	102
Bed silts: Imperial Canals, 1917–18	12	97
Gila River: Silt bars (80)	15	74.2
Deposits in Settling Basins of Imperial Valley Municipal Water Systems:		
El Centro, Calif.	12	45.2
Imperial, Calif.	12	37.0
Calexico, Calif.	10	37.7
Experimental settling basin at Parker City	5	57.5

The writer found the dry weight of all the sediment filtered from the daily samples from Coeur d'Alene River, Idaho, for more than a year (consisting largely of tailings from ore-reducing works) to be 50 lb per cu ft.

If it were not for the uncertain quantity of silt carried into reservoirs as bed load, it would be quite a simple matter to determine the space occupied by the suspended silt in streams flowing into reservoirs where the volume of deposits have been determined by careful capacity surveys. The comparison between suspended silt and deposits in the Elephant Butte Reservoir for determining the probable bed load of the Rio Grande (see heading, "The Bed Load"), shows the futility of this method unless the bed load can be determined by direct measurement in the stream.

As a result of their study of Colorado silts, Fortier and Blaney (42*b*) recommended the use of a specific weight of 62.5 lb per cu ft for suspended silts. This was a convenient figure because the percentage of silt by weight and by volume then becomes equal. For the space occupied in reservoirs on the Lower Colorado River, on account of the probable mixture of coarser bed load and suspended silt and also to allow for compacting by superimposed silt, they suggested the use of a specific weight of silt *in situ* in the reservoir of 85 lb per cu ft (42*c*). As a general average of the silt-bearing streams of Southwestern United States, these figures are consistent with all available data and can be used without great error.

Stream Transportation of Silt

It has been estimated that the surface of the United States is being removed at the rate of 1 in. in 760 yr (3). At some time or other, the material removed is transported by streams. The total of solids removed is estimated at 783 000 000 tons per annum, of which 513 000 000 tons is suspended matter and 270 000 000 tons is dissolved solids.

Flowing water has the power to transport large quantities of finely divided material as a suspended load and also to drag other materials along its bed.

The higher the velocity and the more turbulent the stream, the greater the proportion of suspended load it is capable of carrying. When velocities slacken this material settles and the bed-load movement is arrested. Material remains in suspension by reason of the vertical components of currents and eddies within the water prism.

The shape of the silt particle has an important bearing on the facility with which it remains in suspension or settles to the bottom. The finer the material the slower it settles; rounded particles will settle much faster than flat scale-like particles; and, disk-like particles settle in still water (flat side down) with an oscillating motion requiring a long time.

The velocity of the water, the degree of fineness of the material, and the predominating shape of the particles are three correlated factors that determine the variant between the suspended load and the bed load of a stream.

On mountain streams the bottom load consists of boulders, cobble-stones, and coarse sand. The transporting of stones along a stream bed grinds them, ultimately, into material sufficiently fine to permit their being carried to the ocean as suspended load or as bottom load of slow-moving rivers. By far the greater part of the sediment transportation by streams occurs at the time of a flood. "Cloudbursts" on mountain streams may cause the movement of millions of tons of débris, from fine sands to boulders as large as houses. The waters of great rivers, at their mouths, seldom acquire sufficient velocities to move anything but sand and the smaller gravels.

There is no line of demarcation between suspended load and bed load. In the same stream a given material may be carried in suspension in one reach and as bed load in another. A flood will put in suspension large volumes of material formerly deposited on the bed and banks of the stream and start rolling a new bottom load of much heavier material. The transportation of detritus from mountain top to ocean bed is thus accomplished in stages by a series of spasmodic expenditures of stream energy interspersed with periods of quiescence.

Suspended Silt.—Table 6 gives a summary of the results of a large number of measurements to determine the suspended silt content of the streams of the world. The streams in the United States are arranged alphabetically by the major drainage basins. Tributary streams within those basins are arranged in order from head to mouth.

The Bed Load.—Little information is available as to the quantity of material transported as bed load. Humphreys and Abbot estimated the bed load at the mouth of the Mississippi River to be 11% of the suspended load. Fortier and Blaney place that of the Colorado River, at Yuma, Ariz., at 20% of the total load (42*d*). Follett stated that the bed load of the Rio Grande at San Marcial, N. Mex., "may amount to 25% of the silt carried in suspension."

Comparing the silt content of the Rio Grande during the years between capacity surveys (1916–1925) of Elephant Butte Reservoir, it is found that:

Total suspended silt passing San Marcial. .	=	217 000 000 tons
Total silt deposited in reservoir.	=	178 000 acre-ft.

TABLE 6.—Suspended Silt Carried by Streams

Item No.	Stream	Locality	Drainage area, in square miles	Period	Number of observations in period	Quantity of water, in thousands of acre-feet	Suspended Silt: Per thousand	Suspended Silt: Millions of tons (2000 lb.) during period	Reference number *
				(a) Colorado River System					
1	Colorado.......	Kremmling, Colo.....	2 380	4/23/-12/30, 1905	157	1 030	0.18	0.256	(55)
2	Colorado.......	Palisade, Colo........	8 500	4/2/05-5/5/06	161	4 740	0.49	3.13	(55)
3	Colorado.......	Cisco, Utah..........	24 100	11/1/14-8/31/15	......	6 700	1.0	9.1	(42)
		Cisco, Utah..........	....	Oct-Sept. 1929-30	......	6 150	2.32	19.4	(31)
4	Colorado.......	Lees Ferry, Ariz......	....	Oct-Sept. 1929-30	......	13 200	6.1	110	(31)
				1925-6					
5	Colorado.......	Grand Canyon near	....	1926-7	93	14 400	11.5	225	(62), (58), (61)
		Bright Angel Creek,	....	1927-8	106	17 300	16.9	396	(62), (58), (61)
		Arizona...........	....		201	15 600	8.1	172	(62), (58), (61)
			....	1928-9	259	19 400	18.2	480	(62), (58), (61)
			....	1929-0	289	13 400	13.0	236	(62), (58), (61)
			....	1930-1	291	6 720	7.6	68.8	(62), (58), (61)
			....						
			....	Total mean...	1 239	86 800	13.4	1 580	
6	Colorado.......	Topock, Ariz.........	171 000	8/1/17-7/31/18	......	15 600	9.7	206	(42)
			....	Oct-Sept. 1925-6	98	14 300	7.2	140	(62), (58), (61)
			....	1926-7	104	17 000	14.9	345	(62), (58), (61)
			....	1927-8	103	15 400	10.0	209	(62), (58), (61)
			....	1928-9	98	18 900	17.1	437	(62), (58), (61)
			....	1929-0	102	13 200	13.6	245	(62), (58), (61)
			....	1930-1	94	6 770	7.0	65	(62), (58), (61)
			....						
			....	Total mean..	599	85 570	12.4	1 441	
7	Colorado.......	Yuma, Ariz..........	242 000	Jan-Dec., 1911	......	17 800	11.3	273	(50c)
			....	1912	......	18 400	6.5	163	(50c)
			....	1913	......	11 800	7.4	119	(50c)
			....	1914	......	20 700	9.3	261	(50)
			....	1915	......	14 600	11.6	230	(50)
			....	1916	......	22 900	14.2	442	(50)
			....	1917	......	20 600	5.9	165	(50)
			....	1918	......	13 100	6.3	112	(50)
			....	1919	......	10 700	10.0	145	(50)
			....	1920	......	21 400	8.6	251	(50)
			....	1921	......	19 400	8.6	228	(50)
			....	1922	......	17 000	8.2	190	(50)
			....	1923	......	17 800	10.4	252	(50)
			....	1924	......	11 400	8.3	129	(50)
			....	1925	......	12 400	9.6	162	(50)
			....	1926	......	12 200	7.6	126	(50)
			....	1927	......	17 100	10.6	245	(50)
			....	1928	......	12 800	6.6	116	(58)
			....	1929	......	17 500	12.4	292	(58)
			....	1930	......	10 600	12.9	186	(58)
			....	1931	......	4 800	8.1	53	(58)
			....						
			....	Total mean..	......	325 000	9.4	4 140	
8	Gunnison.......	Whitewater, Colo.....	7 870	4/2/-10/31, 1905	203	2 290	0.64	2.01	(55)
9	Green..........	Green River, Wyo....	7 450	5/1-11/1, 1905	145	935	0.10	0.132	(55)
10	Green..........	Green River, Utah....	40 600	8/1/14-8/31/15	......	4 700	1.3	8.3	(42)
11	San Juan.......	Bluff, Utah..........	24 000	11/1/14-8/31/15	......	3 000	4.1	16.6	(42)
		Bluff, Utah..........	....	Oct-Sept. 1929-30	......	1 740	19.3	45.6	(31)
12	Animas........	Durango, Colo........	810	3/19-12/18, 1905	188	850	0.18	0.210	(55)
13	Little Colorado..	Woodruff, Ariz.......	6 000	8/6/05-4/3/06	126	85.5	7.86	0.915	(55)
14	Gila...........	Florence Canal.......	....	11/28/99-3/7/1900	......	....	0.38	....	(42)
		Florence Canal.......	....	8/1-11/5, 1900	......	....	26.8	....	(42)
15	Gila...........	San Carlos, Ariz......	13 500	6/1-12/31, 1905	......	255	4.9	1.7	(42)
16	Gila...........	Yuma, Ariz..........	....	8/5-10/15, 1914	......	74.6	53.0	5.4	(42)
		Yuma, Ariz..........	....	11/11-12/3, 1916	......	54.5	5.4	0.4	(42)
17	San Francisco...	Alma, N. Mex........	1 800	4/14/05-4/22/06	161	165	1.76	0.396	(55)
18	Salt...........	Roosevelt, Ariz.......	5 760	4/9/05-4/23/06	155	4 280	2.01	11.7	(55)
19	Verde..........	McDowell, Ariz.......	6 000	4/5/05-3/10/06	143	615	1.23	1.03	(55)

* See Appendix.

TABLE 6—(*Continued*)

Item No.	Stream	Locality	Drainage area, in square miles	Period	Number of observations in period	Quantity of water, in thousands of acre-feet	Suspended Silt: Per thousand	Suspended Silt: Millions of tons (2000 lb.) during period	Reference number *
				(*b*) Columbia River System					
20	No. Fork, Coeur d'Alene	Enaville, Idaho		5/13/21–6/30/22	414	1 440	0.074	0.146	(35)
21	So. Fork, Coeur d'Alene	Enaville, Idaho		5/13/21–6/30/22	414	466	0.97	0.615	(35)
22	Salmon	Malott, Wash.	150	5/23/05–1/13/06	139	25.5	0.067	0.0023	(55)
23	Malheur	Vale, Ore.	4 860	3/26–12/4, 1905	186	134	0.193	0.035	(55)
24	Payette	Horseshoe Bend, Idaho	2 240	5/15–9/13, 1906	75	1 030	0.031	0.043	(55)
25	Palouse	Hooper, Wash.	2 210	5/22–10/8, 1905	122	32.2	0.055	0.0024	(55)
				(*c*) Great Basin River Systems					
26	Truckee	Derby, Nev.	1 750	4/10/06–3/13/07	39	1 130	0.053	0.081	(55)
27	Owens	Round Valley, Calif.	400	5/13/06–3/30/07	290	232	0.026	0.0081	(55)
				(*d*) Mississippi River System					
28	Mississippi	Prescott, Wis.		4/26–7/30, 1881	14	12 800	0.161	2.79	(38)
29	Mississippi	Winona, Minn.		2/4–7/30, 1881	28	13 100	0.033	0.59	(38)
30	Mississippi	Winona, Minn.		May-Oct. 1932	.....	....	......	0.598	(69)
31	Mississippi	LaCrosse, Wis.		Apr.-Nov. 1932	.....	....	......	1.044	(69)
32	Mississippi	Clayton, Iowa		1/28–8/18, 1881	38	21 800	0.039	1.16	(38)
33	Mississippi	Clayton, Iowa		May-Oct. 1932	.....	....	......	1.673	(69)
34	Mississippi	Hannibal, Mo.		1/11–8/4, 1881	48	54 000	0.28	20.5	(38)
35	Mississippi	Grafton, Ill.		11/13/80–8/31/81	89	105 000	0.321	45.8	(38)
		Grafton, Ill.		3/22–6/6, 1929	12	41 000	0.256	14.2	(38)
		Grafton, Ill.	170 000	6/6/30–/2/28/31	36	20 900	0.098	2.79	(39)
36	Mississippi	St. Louis, Mo.		3/31–6/25, 1879	21	29 800	2.44	99.0	(38)
		St. Louis, Mo.		1/15–9/5, 1881	36	118 000	1.40	225	(38)
		St. Louis, Mo.		4/8–6/12, 1929	8	62 000	1.54	132	(38)
37	Mississippi	Columbus, Ky.		3/15–11/15, 1858	146	262 000	0.83	293	(38)
		Columbus, Ky.		3/4–7/2, 1879	79	376 000	1.43	177	(38)
38	Misssissippi	Fulton, Tenn.		11/28/79–10/10/80	178	355 000	0.88	424	(38)
39	Mississippi	Hampton Ldg., Ark.		1/6–6/27, 1879	63	157 000	0.63	135	(38)
40	Mississippi	Helena, Ark.		12/13/78–6/18/79	37	212 000	0.65	188	(38)
		Helena, Ark.	938 000	9/2/30–2/28/31	61	55 000	0.35	26.4	(39)
41	Mississippi	Chicot, Ark.		4/2–6/25, 1929	25	240 000	0.44	142	(38)
		Chicot, Ark.	1 119 000	9/2/30–1/17/31	80	40 500	0.53	29.5	(39)
42	Mississippi	Lake Providence, La.		11/18/79–10/15/80	28	355 000	0.70	335	(38)
43	Mississippi	Kings Point, Miss.		1/17–5/30, 1879	49	195 000	0.57	152	(38)
44	Mississippi	Vicksburg, Miss.		3/13–6/6, 1929	16	235 000	0.55	177	(38)
		Vicksburg, Miss.	1 138 000	8/29/30–1/26/31	63	45 000	0.45	27.0	(39)
45	Mississippi	Tarberts Ldg., Miss.		3/19–6/21, 1929	25	263 000	0.39	140	(38)
46	Mississippi	Red River Ldg., Miss.		3/3–6/22, 1929	25	249 000	0.37	126	(38)
		Red River Ldg., Miss.	1 230 000	9/23/30–2/26/31	65	60 500	0.57	46.6	(39)
47	Mississippi	Carrollton, La.		2/17/51–2/15/52	52	452 000	0.63	380	(38)
		Carrollton, La.		2/16/52–2/20/53	32	530 000	0.81	582	(38)
		Carrollton, La.		12/19/79–10/8/80	29	304 000	0.71	293	(38)
		Carrollton, La.		3/12–6/25, 1929	39	227 000	0.80	246	(38)
		Carrolton, La.	1 238 000	9/16/30–2/27/31	65	63 000	0.26	22.0	(39)
48	Chippewa	Durand, Wis.		May-Oct., 1932	.....	....	......	0.050	(69)
49	LaCrosse	West Salem, Wis.		May-Oct., 1932	.....	....	......	0.071	(69)
50	Root	Huston, Minn.		May-Oct., 1932	.....	....	......	0.687	(69)
51	Wisconsin	Musooda, Wis.		May-Oct., 1932	.....	....	......	0.135	(69)
52	Illinois River	Lockport (Sta. 292)	217	Sept-Aug., 1921–22	.....	6 400	0.032	0.264	(59)
53	Illinois River	Joliet (Sta. 286)	1 120	Sept-Aug. 1921–22	.....	6 560	0.048	0.430	(59)
54	Illinois River	Morris (Sta. 263)	7 600	Sept-Aug. 1921–22	.....	10 800	0.082	1.20	(59)
55	Illinois River	Chillicothe (Sta. 179)	13 400	Sept-Aug. 1921–22	.....	13 900	0.039	0.750	(59)
56	Illinois River	Peoria (Sta. 166)	13 600	Sept-Aug. 1921–22	.....	14 000	0.030	0.565	(59)
57	Missouri	Ft. Benton, Mont.	24 600	July-June, 1929–30	†433	4 300	0.077	0.45	(43)
		Ft. Benton, Mont.		1930–31	.....	3 120	0.177	0.749	(43)
58	Missouri	Williston, N. Dak.	164 000	1929–30	†568	14 600	1.42	28.1	(43)
		Williston, N. Dak.		1930–31	.....	11 500	2.28	35.6	(43)
59	Missouri	Mobridge, S. Dak.	209 000	1929–30	†292	16 300	2.10	46.6	(43)
		Mobridge, S. Dak.		1930–31	.....	10 900	2.38	35.3	(43)
60	Missouri	Pierre, S. Dak.	244 000	1929–30	†220	17 400	3.37	79.7	(43)
		Pierre, S. Dak.		1930–31	.....	11 400	2.38	36.9	(43)
61	Missouri	Sioux City, Iowa	315 000	1929–30	†163	19 800	3.83	103	(43)
		Sioux City, Iowa		1930–31	.....	13 000	2.14	38.1	(43)
62	Missouri	Omaha, Nebr.	323 000	1929–30	†630	20 500	4.05	113	(43)
		Omaha Nebr.		1930–31	.....	13 100	2.48	44.1	(43)
		Omaha, Nebr.		1931–32	.....	16 800	4.82	110	(43)

* See Appendix. † Number of observations for entire period.

TABLE 6—(*Continued*)

Item No.	Stream	Locality	Drainage area, in square miles	Period	Number of observations in period	Quantity of water, in thousands of acre-feet	Suspended Silt: Per thousand	Suspended Silt: Millions of tons (2000 lb.) during period	Reference number *
				(d) Mississippi River System (*Continued*)					
63	Missouri	Plattsmouth, Nebr.	414 000	1929–30	†490	26 400	3.93	141	(43)
		Plattsmouth, Nebr.		1930–31		18 300	2.07	51.4	(43)
		Plattsmouth, Nebr.		1931–32		21 000	4.30	123	(43)
64	Missouri	Leavenworth, Kans.	425 000	1929–30	†648	28 900	4.20	170	(43)
		Leavenworth, Kans.		1930–31		18 900	2.43	62.3	(43)
65	Missouri	Kansas City, Mo.	489 000	1929–30	†923	34 000	4.13	191	(43)
		Kansas City, Mo.		1930–31		22 000	2.37	70.8	(43)
		Kansas City, Mo.		1931–32		30 800	4.98	208	(43)
66	Missouri	Boonville, Mo.	506 000	1929–30	†499	39 400	3.57	191	(43)
		Boonville, Mo.		1930–31		24 800	2.01	67.6	(43)
67	Missouri	Howard Bend, Mo.	529 000	1929–30	†988	43 800	3.24	193	(43)
		Howard Bend, Mo.		1930–31		27 800	2.05	77.3	(43)
		Howard Bend, Mo.		1931–32		44 900	3.00	183	(43)
68	Missouri	St. Charles, Mo.		2/1–10/31, 1879	273	45 500	4.10	253	(38)
		St. Charles, Mo.	528 000	8/28/30–2/28/31	44	13 500	2.56	47.0	(39)
69	Marias	Loma, Mont.	9 160	July–June, 1929–30	†324	589	0.467	0.374	(43)
		Loma, Mont.		1930–31		355	0.578	0.279	(43)
70	Musselshell	Mosby, Mont.	9 570	1929–30	†157	75.2	1.13	0.115	(43)
		Mosby, Mont.		1930–31		19.5	6.26	0.166	(43)
71	Milk	Nashau, Mont.	23 800	1929–30	†320	405	1.91	1.05	(43)
		Nashau, Mont.		1930–31		62.9	0.171	0.0146	(43)
72	Yellowstone	Billings, Mont.	11 180	5/20–11/24, 1905	88	3 560	0.43	2.07	(55)
73	Yellowstone	Glendive, Mont.	66 100	3/28/05–4/21/06	172	7 570	1.14	11.8	(55)
		Glendive, Mont.	66 900	July–June, 1929–30	†380	8 480	2.16	24.9	(43)
		Glendive, Mont.		July–June, 1930–31		7 240	3.64	35.8	(43)
74	Big Horn	Fort Custer, Mont.	20 700	6/10/05–6/9/06	72	4 360	1.23	7.3	(55)
75	Shoshone	Cody, Wyo.	1 480	4/1/05–3/31/06	287	1 020	0.121	0.168	(55)
76	Little Missouri	Medora, N. Dak.	6 320	July–June, 1929–30	†387	208	5.80	1.64	(43)
		Medora, N. Dak.		1930–31		80.6	8.98	0.984	(43)
77	Cannonball	Timmer, N. Dak.	3 660	1929–30	†121	77.1	7.29	0.764	(43)
		Timmer, N. Dak.		1930–31		16.8	2.07	0.0473	(43)
78	Grand	Wakpala, S. Dak.	5 660	1929–30	†150	105	5.76	0.822	(43)
		Wakpala, S. Dak.		1930–31		44.1	3.62	0.217	(43)
79	Moreau	Promise, S. Dak.	5 200	1929–30	†121	87.0	6.17	0.730	(43)
		Promise, S. Dak.		1930–31		42.8	6.24	0.363	(43)
80	Cheyenne	Carlin, S. Dak.	25 500	1929–30	†546	720	8.23	8.05	(43)
		Carlin, S. Dak.		1930–31		410	10.86	6.05	(43)
81	Belle Fourche	Belle Fourche, S. Dak.	3 250	4/15–11/25, 1905	192	121	3.60	0.590	(55)
		Belle Fourche, S. Dak.		4/1–6/23, 1906	51	101	2.71	0.372	(55)
		Belle Fourche, S. Dak.	4 270	7/27–11/13, 1906	89	53.7	0.68	0.050	(55)
82	Redwater	Belle Fourche, S. Dak.	1 020	4/9–11/25, 1905	188	159	0.26	0.057	(55)
		Belle Fourche, S. Dak.		4/1–6/23. 1906	53	48.5	0.42	0.028	(55)
83	Bad	Fort Pierre, S. Dak.	3 110	July–June, 1929–30	†142	87.6	38.97	4.64	(43)
		Fort Pierre, S. Dak.		1930–31		106	38.32	5.52	(43)
84	White	Oacoma, S. Dak.	10 200	1929–30	†227	432	22.99	13.5	(43)
		Oacoma, S. Dak.		1930–31		278	18.21	6.88	(43)
85	Niobrara	Verdel, Nebr.	12 300	1929–30	†637	1 110	0.628	0.947	(43)
		Verdel, Nebr.		1930–31		970	0.459	0.605	(43)
86	James	Scotland, S. Dak.	21 500	1929–30	†261	112	0.168	0.0255	(43)
		Scotland, S. Dak.		1930–31		46.5	0.118	0.00747	(43)
87	Big Sioux	Akron, Iowa	9 420	1929–30	†374	358	0.742	0.361	(43)
		Akron, Iowa		1930–31		92.2	0.173	0.0217	(43)
88	Little Sioux	Correctionville, Iowa	4 260	1929–30	†376	202	1.59	0.437	(43)
		Correctionville, Iowa		1930–31		40.2	0.198	0.0108	(43)
89	Platte	Duncan, Nebr.	66 100	1929–30	†174	2 150	0.657	1.92	(43)
		Duncan, Nebr.		1930–31		1 870	0.504	1.28	(43)
90	Platte	Plattsmouth, Nebr.	90 200	1929–30	†886	5 010	2.47	16.8	(43)
		Plattsmouth, Nebr.		1930–31		4 440	1.55	9.34	(43)
		Plattsmouth, Nebr.		1931–32		3 780	1.48	7.61	(43)
91	North Platte	Laramie, Wyo.	16 200	5/21–12/28, 1906	125	1 410	0.99	1.90	(55)
92	Loup	Genoa, Nebr.	13 600	July–June, 1929–30	†208	1 970	2.89	7.74	(43)
		Genoa, Nebr.		1930–31		1 930	1.89	4.96	(43)
93	Elkhorn	Waterloo, Nebr.	6 560	1929–30	†191	744	3.19	3.32	(43)
		Waterloo, Nebr.		1930–31		586	0.662	0.527	(43)
94	Kansas	Bonner Springs, Kans.	61 300	1929–30	†617	3 830	3.94	20.5	(43)
		Bonner Springs, Kans.		1930–31		2 780	2.19	8.28	(43)
95	Kansas	Holliday, Kans.	62 000	Jan–Dec., 1907	365	3 820	0.592	4.48	(8a)
		Holliday, Kans.		1908	365	9 500	0.895	23.3	(8a)

* See Appendix. † Number of observations for entire period.

TABLE 6—(*Continued*)

Item No.	Stream	Locality	Drainage area, in square miles	Period	Number of observations in period	Quantity of water, in thousands of acre-feet	Suspended Silt: Per thousand	Suspended Silt: Millions of tons (2000 lb.) during period	Reference number *
				(d) Mississippi River System (*Continued*)					
96	Smoky Hill	Mentor, Kans.	8 420	July-June, 1929–30	†209	270	3.50	1.29	(43)
		Mentor, Kans.	....	1930–31	.....	224	1.79	0.544	(43)
97	Smoky Hill	Solomon, Kans.	19 200	1929–30	†368	794	2.52	2.72	(43)
		Solomon, Kans.	....	1930–31	.....	609	1.49	1.23	(43)
98	Saline	Tescott, Kans.	2 880	July-June, 1929–30	†244	79.8	1.76	0.191	(43)
		Tescott, Kans.	....	1930–31	.....	142	1.97	0.381	(43)
99	Solomon	Niles, Kans.	6 900	1929–30	†259	255	3.12	1.08	(43)
		Niles, Kans.	....	1930–31	.....	198	2.99	0.805	(43)
100	Republican	Wakefield, Kans.	25 300	July-June, 1929–30	†543	722	5.61	5.50	(43)
		Wakefield, Kans.	....	1930–31	.....	664	3.97	3.58	(43)
101	Big Blue	Randolph, Kans.	9 360	1929–30	†407	1 000	4.25	5.77	(43)
		Randolph, Kans.	....	1930–31	.....	645	2.30	2.02	(43)
102	Grand	Gallatin, Mo.	2 250	1929–30	†53	611	3.93	3.26	(43)
		Gallatin, Mo.	....	1930–31	.....	166	2.34	0.527	(43)
103	Grand	Sumner, Mo.	6 880	1929–30	†484	1 960	3.05	8.12	(43)
		Sumner, Mo.	....	1930–31	.....	959	2.63	3.43	(43)
104	Thompson	Trenton, Mo.	1 670	1929–30	†54	438	8.27	4.92	(43)
		Trenton, Mo.	....	1930–31	.....	134	4.87	0.887	(43)
105	Osage	Bagnell, Mo.	14 000	1929–30	†395	3 070	0.338	1.41	(43)
		Bagnell, Mo.	....	1930–31	.....	877	0.080	0.0955	(43)
106	Gasconade	Rich Fountain, Mo.	3 180	1929–30	†166	1 660	0.081	0.183	(43)
		Rich Fountain, Mo.	....	1930–31	.....	1 150	0.070	0.110	(43)
107	Ohio	Paducah, Ky.	....	12/16/78–12/30/79	76	174 000	0.32	76.0	(38)
108	Ohio	Mound City, Ill.	202 000	9/11/30– 2/27/31	66	23 000	0.12	3.75	(39)
109	White	Clarendon, Ark.	....	1/19–6/26, 1879	27	6 200	0.036	0.30	(38)
110	White	DeValls Bluff, Ark.	23 500	2/6–5/30, 1931	36	7 100	0.113	1.09	(39)
111	Arkansas	Pine Bluff, Ark.	....	2/20–7/8, 1879	134	4 110	0.51	2.88	(38)
112	Arkansas	Tulsa, Okla.	74 700	10/18/30–9/8/31	73	2 400	4.15	13.55	(39)
113	Arkansas	Ozark, Ark.	152 000	10/22/30–9/2/31	94	13 200	1.81	32.4	(39)
114	Cimarron	Guthrie, Okla.	16 000	10/15/30–9/8/31	68	532	8.00	5.80	(39)
115	Verdigris	Okay, Okla.	8 140	10/28/30–9/7/31	74	1 840	1.12	2.80	(39)
116	Grand	Wagoner, Okla.	12 400	10/24/30–9/7/31	73	3 150	0.32	1.38	(39)
117	So. Canadian	Calvin, Okla.	29 700	10/30/30–9/2/31	71	450	5.70	3.46	(39)
118	Yazoo	Greenwood, Miss.	7 700	9/16/30–9/26/31	122	3 960	0.43	2.35	(39)
119	Ouachita	Monroe, La.	17 760	9/17/30–9/29/31	179	910	0.13	0.16	(39)
120	Red	Denison, Tex.	36 100	9/9/30–9/30/31	160	2 950	3.73	15.0	(39)
121	Red	Alexandria, La.	....	2/24–7/1, 1879	22	6 550	0.38	3.40	(38)
		Alexandria, La.	63 300	9/23/30–9/19/31	156	16 600	1.56	35.2	(39)
122	Salt Fk. of Red R.	Mangum, Okla.	1 220	4/11/05–6/28/06	253	63.5	2.36	0.204	(55)
123	North Fk. of Red R.	Granite, Okla.	2 210	4/12/05–3/16/07	482	425	4.21	2.30	(55)
124	North Fk. of Red R.	Headrick, Okla.	5 470	5/20/05–3/19/07	447	700	3.45	3.28	(55)
125	Elm Fk. of Red R.	Mangun, Okla.	750	5/13/05–3/22/07	509	350	7.02	3.34	(55)
126	Washita	Durwood, Okla.	....	9/8/30–9/23/31	109	785	4.40	4.70	(39)
127	Little River	Horatio, Ark	....	1/21–9/28, 1931	85	1 670	0.053	0.12	(39)
128	Sulphur	Darden, Tex.	....	9/10/30–3/24/31	125	1 010	0.65	0.885	(39)
129	Atchafalaya	Simmesport, La.	....	3/19–6/22, 1929	23	63 000	0.38	32.6	(38)
		Simmesport, La.	....	9/23/30–2/27/31	60	11 700	1.18	18.7	(38)
				(e) Gulf of Mexico Drainage System					
130	Rio Grande	San Marcial, N. Mex.	30 000	Jan-Dec., 1897	.....	2 220	14.5	43.7	(47), (65)
			....	1898	.....	961	13.1	17.0	(48)
			....	1899	.....	239	18.1	5.9	(48)
			....	1900	.....	468	17.1	10.9	(48)
			....	1901	.....	656	23.8	21.3	(48)
			....	1902	.....	201	25.8	7.0	(48)
			....	1903	.....	1 270	8.2	14.2	(48)
			....	1904	.....	710	20.0	19.3	(48)
			....	1905	35	2 420	6.6	21.8	(48)
			....	1906	112	1 560	7.5	16.0	(48)
			....	1907	118	2 160	9.4	27.5	(48)
			....	1908	110	774	16.9	17.8	(48)
			....	1909	117	1 280	12.9	22.2	(48)
			....	1910	90	852	6.4	7.5	(48)
			....	1911	114	1 800	35.0	85.2	(48)
			....	1912	109	1 500	12.4	25.3	(48)

* See Appendix. † Number of observations for entire period.

TABLE 6—(*Continued*)

Item No.	Stream	Locality	Drainage area, in square miles	Period	Number of observations in period	Quantity of water, in thousands of acre-feet	Suspended Silt: Per thousand	Suspended Silt: Millions of tons (2000 lb.) during period	Reference number *
				(*e*) Gulf of Mexico Drainage System (*Continued*)					
130	Rio Grande	San Marcial, N. Mex.	30 000	Jan-Dec., 1913	.....	525	8.2	5.8	(58)
			....	1914	.....	1 180	25.3	40.5	(58)
			....	1915	.....	1 350	13.6	25.0	(58)
			....	1916	.....	1 650	13.2	30.0	(58)
			....	1917	.....	1 050	6.6	9.4	(58)
			....	1918	.....	410	6.6	3.7	(58)
			....	1919	.....	1 580	22.8	45.8	(58)
			....	1920	.....	2 220	10.4	31.5	(58)
			....	1921	.....	1 630	19.2	42.9	(58)
			....	1922	.....	964	10.2	13.3	(58)
			....	1923	.....	1 220	11.7	19.4	(58)
			....	1924	.....	1 440	7.5	14.6	(58)
			....	1925	.....	419	12.0	6.8	(58)
			....	1926	.....	1 050	7.2	10.2	(58)
			....	1927	.....	1 350	19.6	35.9	(58)
			....	1928	.....	590	5.7	4.6	(58)
			....	1929	.....	1 460	29.5	58.6	(58)
			....	1930	.....	731	5.4	5.4	(58)
			....	1931	.....	490	18.8	12.5	(58)
			Total mean, Item No. 130		.....	40 400	14.2	780.0	
131	Rio Grande.....	El Paso, Tex.........	....	6/1/89–8/31/90	297	1 075	4.0	5.86	(48)
		El Paso, Tex.........	38 600	1/8/05–4/30/07	248	2 750	8.1	30.3	(55)
		El Paso, Tex.........	‡32 800	Jan-Dec., 1924	26	816	1.67	1.91	(14)
132	Rio Grande.....	Fort Quitman, Tex....	‡34 500	Jan-Dec., 1924	25	505	3.57	2.46	(14)
133	Rio Grande.....	Upper Presidio, Tex...	‡37 500	4/23–12/29, 1924	19	311	2.30	0.965	(14)
		Upper Presidio, Tex...	....	Jan-Dec., 1925	26	407	3.58	2.04	(14)
		Upper Presidio, Tex...	....	1926	26	570	4.06	3.26	(14)
134	Rio Grande.....	Lower Presidio, Tex...	‡60 100	4/24–12/30, 1924	19	780	3.03	2.18	(14)
		Lower Presidio, Tex...	....	Jan-Dec., 1925	26	2 720	3.50	13.1	(14)
		Lower Presidio, Tex...	....	1926	26	2 590	3.11	11.0	(14)
135	Rio Frande.....	Boquillas, Tex........	‡69 400	5/7–10/17, 1929	46	675	9.75	8.9[illegible]	(14)
136	Rio Grande.....	Laredo, Tex..........	‡133 000	6/13–12/16, 1924	17	3 360	3.85	17.4	(14)
		Laredo, Tex..........	....	Jan-Dec., 1925	29	7 250	3.40	24.6	(14)
		Laredo, Tex..........	....	1926	24	5 690	1.93	14.8	(14)
137	Rio Grande.....	Roma, Tex...........	‡160 000	6/16–12/25, 1924	15	4 060	4.40	24.1	(14)
		Roma, Tex...........	....	Jan-Dec., 1925	25	8 040	7.40	85.2	(14)
		Roma, Tex...........	....	1/11–10/25, 1926	17	5 550	7.62	45.6	(14)
		Roma, Tex...........	....	3/6–12/31, 1929	296	2 320	2.81	8.91	(14)
			....	Jan-Dec., 1930	257	3 400	3.76	17.3	(14)
		Roma, Tex..........		1931	257	3 080	1.87	7.82	(14)
138	Rio Grande.....	Matamoras, Mex.	‡180 000	4/17–12/22, 1924	20	2 620	2.99	10.6	(14)
		Matamoras, Mex.....	....	Jan-Dec., 1925	28	5 630	3.35	25.7	(14)
		Matamoras, Mex.....	....	1926	27	5 740	3.68	28.8	(14)
139	Pecos..........	Santa Rosa, N. Mex..	2 900	7/7/05–12/27/07	380	191	5.5	1.43	(55)
140	Pecos..........	Dayton, N. Mex......	20 000	7/20/05–4/20/07	417	790	5.8	6.22	(55)
141	Pecos..........	Carlsbad, N. Mex.....	22 000	5/22/05—4/30/07	366	1 110	0.53	0.80	(55)
142	Gallinas.........	La Vegas, N. Mex....	90	5/19/05–4/31/06	262	28.3	0.05	0.0019	(55)
143	Hondo.........	Roswell, N. Mex......	1 040	4/26–8/4, 1905	96	78 0	9.80	1.04	(55)
144	Brazos.........	Mineral Wells, Tex....	23 100	Oct-Sept., 1924–27	.....	2 940	10.1	40.5	(53)
145	Brazos.........	Waco, Tex...........	28 500	Oct-Sept., 1924–27	.....	4 890	8.6	57.2	(53)
146	Brazos.........	College Sta., Tex.....	37 400	8/1–12/31, 1899	7	1 160	8.66	....	(34), (33)
		College Sta., Tex......	....	Jan-Dec., 1900	31	8 810	13.15	....	(34), (33)
		College Sta., Tex.....	....	1901	60	977	12.62	....	(34)
147	Brazos.........	Rosenburg, Tex.......	44 000	Oct-Sept., 1924–27	.....	15 600	4.3	106	(53)
148	Double Mt. Fk. of Brazos R...	Aspermont, Tex......	7 980	Oct-Sept., 1924–27	.....	510	21.3	13.9	(53)
149	Witchita.......	Witchita Falls, Tex...	3 050	2/10–12/31,1900	20	842	12.07	....	(34), (33)
		Witchita Falls, Tex...	....	Jan-Dec., 1901	52	298	15.57	....	(34)
				(*f*) Sacramento River System					
150	Sacramento.....	Red Bluff, Calif......	9 300	7/3/05–3/23/07	446	20 800	0.107	3.02	(55)
151	Pit...........	Bieber, Calif.........	2 950	7/7/05–3/2/07	285	924	0.106	0.133	(55)
152	Feather........	Oroville, Calif.......	3 640	6/25/05–2/14/07	379	9 600	0.098	1.26	(55)
153	Puta Cr........	Winters, Calif........	805	1/2/06–3/1/07	371	873	0.43	0.507	(55)

* See Appendix. ‡ With all closed basins eliminated.

TABLE 6—(*Continued*)

Item No.	Stream	Locality	Drainage area, in square miles	Period	Number of observations in period	Quantity of water, in thousands of acre-feet	Suspended Silt: Per thousand	Suspended Silt: Millions of tons (2000 lb.) during period	Reference number *
				(g) Nile River System					
154	Nile	Sarras, Egypt	....	6/22–8/20, 1905	27	9 650	0.614	8.04	(23)
		Sarras, Egypt	....	7/7–8/20, 1906	19	25 300	1.97	67.6	(23)
155	Nile	Aswan, Egypt	....	Jan.	.....	3 500	0.25	1.20	(9)
		Aswan, Egypt	....	Feb.	.....	2 600	0.17	0.60	(9)
		Aswan, Egypt	....	Mar.	.....	1 960	0.10	0.27	(9)
		Aswan, Egypt	....	April	.....	1 500	0.08	0.16	(9)
		Aswan, Egypt	....	May	.....	1 280	0.07	0.12	(9)
		Aswan, Egypt	....	June	.....	1 600	0.10	0.22	(9)
		Aswan, Egypt	....	July	.....	4 850	0.15	1.00	(9)
		Aswan, Egypt	....	Aug.	.....	17 000	1.43	33.0	(9)
		Aswan, Egypt	....	Sept.	.....	19 200	1.32	34.0	(9)
		Aswan, Egypt	....	Oct.	.....	13 300	0.83	15.0	(9)
		Aswan, Egypt	....	Nov.	.....	7 000	0.57	5.4	(9)
		Aswan, Egypt	....	Dec.	.....	4 650	0.36	2.29	(9)
			....	Mean year	.....	78 440	0.87	93.26	
		Aswan, Egypt	....	Jan–Dec., 1913	51	35 500	0.53	25.2	(36); (51)
		Aswan, Egypt	....	1914	52	63 900	1.20	104.0	(36); (51)
		Aswan, Egypt	....	1915	52	54 500	0.66	48.5	(36); (51)
			....	1916	52	85 600	0.85	98.7	(36); (51)
			....	1917	52	91 500	0.68	83.6	(36); (51)
			....	1918	52	67 600	0.52	48.2	(36); (51)
			....	1919	50	60 300	0.99	81.2	(36); (51)
			....	1920	52	67 400	0.85	78.0	(36); (51)
			....	1921	50	59 700	0.92	74.5	(36); (51)
			....	1922	51	72 000	1.02	100.0	(36); (51)
			....	1923	52	72 700	0.86	84.8	(36); (51)
			....	1924	52	75 500	0.87	89.3	(36); (51)
			....	1925	52	57 200	0.69	53.3	(36); (51)
			....	1926	52	69 000	0.90	84.2	(36); (51)
			....	14 years, total mean	722	932 000	0.83	1 050	
				(h) Other Streams in Africa					
160	Orange	Orange R. Sta., C. P.	....	Jan–April, 1920	.....	5 990	6.9	534	(56a)
			....	3/17–5/31, 1921	.....	2 100	4.4	13.2	(56a)
161	Lower Orange	Kakamas	....	11/18/19–5/5/20	26	12 000	8.70	142	(56b)
162	Vaal	Kimberley	....	1/1/18–12/10/19	21	14 100	1.88	36	(56c)
				(i) Tigris River, Asia					
163	Tigris	Amara, Irak	....	Jan., 1918	.....	0.60	4.1	3.00	(49)
			....	Feb.	.....	0.51	0.68	0.42	(49)
			....	March	.....	1.08	3.77	4.92	(49)
			....	April	.....	1.11	2.52	3.51	(49)
			....	May	.....	1.18	2.03	2.89	(49)
			....	June	.....	0.86	0.60	0.63	(49)
			....	July	.....	0.59	0.35	0.26	(49)
			....	Aug.	.....	0.43	0.21	0.11	(49)
			....	Sept.	.....	0.30	0.15	0.05	(49)
			....	Oct.	.....	0.28	0.12	0.04	(49)
			....	Nov.	.....	0.36	0.18	0.08	(49)
			....	Dec.	.....	0.62	0.77	0.59	(49)
			....	The year	.....	7.92	1.54	16.50	
				(j) Streams in India					
164	Kitsna	Bezwada, India	97 000	6/22–11/30, 1898	.....	51 800	2.78	196	(21)
			....	6/22–11/9, 1899	.....	19 300	5.26	138	(21)
			....	6/22–11/19, 1900	.....	56 500	3.41	262	(21)
			....	6/1–7/31, 1901	.....	15 000	5.14	105	(21)
			....	6/8–11/15, 1909	.....	42 100	3.08	176	(21)
			....	Total mean	.....	184 700	3.50	877	
165	Indus	Sukkur, India	....	Jan–Dec., 1902	.....	84 200	2.70	319	(20)
			....	1903	.....	116 000	3.14	495	(20)

* See Appendix.

TABLE 6—(*Continued*)

Item No.	Stream	Locality	Drainage area, in square miles	Period	Number of observations in period	Quantity of water, in thousands of acre-feet	SUSPENDED SILT: Per thousand	SUSPENDED SILT: Millions of tons (2000 lb.) during period	Reference number *
				(*j*) STREAMS IN INDIA (*Continued*)					
165	Indus..........	Sukkur, India........	....	1904	.....	102 000	2.69	373	(20)
			....	1905	.....	121 000	2.86	472	(20)
			....	1906	.....	113 000	3.68	566	(20)
			....	1907	.....	99 000	2.60	349	(20)
			....	1908	.....	136 000	3.04	562	(20)
			....	1909	.....	132 000	3.46	623	(20)
			....	1910	.....	129 000	4.02	708	(20)
			....	1911	.....	142 000	3.60	697	(20)
			....	1912	.....	127 000	3.56	615	(20)
			....	1913	.....	105 000	2.75	394	(20)
			....	1914	.....	159 000	2.93	635	(20)
			....	1915	.....	129 000	2.83	495	(20)
			....	1916	.....	118 000	2.80	448	(20)
			....	1917	.....	126 000	3.57	613	(20)
			....	1918	.....	116 000	2.86	450	(20)
			....	1919	.....	110 000	2.34	350	(20)
			....	1920	.....	106 000	2.77	400	(20)
			....	1921	.....	98 000	2.76	367	(17)
			....	1922	.....	119 000	2.80	454	(17)
			....	1923	.....	118 000	2.98	478	(17)
			....	1924	.....	127 000	2.66	459	(17)
			....	1925	.....	107 000	2.68	390	(17)
			24 years, total	mean Item No. 165	.....	2 840 000	3.03	11 712	
166	Indus..........	Kotri, India.........	....	Jan-Dec., 1902	.....	73 500	3.02	304	(20)
			....	1903	.....	98 000	3 56	474	(20)
			....	190	.....	84 500	3.22	371	(20)
			....	1905	.....	106 000	3.12	450	(20)
			....	1906	.....	105 000	3.48	516	(20)
			....	1907	.....	206 000	1.27	356	(20)
			....	1908	.....	115 000	2.74	428	(20)
			....	1909	.....	106 000	3.08	444	(20)
			....	1910	.....	112 000	2.88	438	(20)
			....	1911	.....	122 000	3.38	563	(20)
			....	1912	.....	86 500	2.76	325	(20)
			....	1913	.....	84 700	2.76	318	(20)
			....	1914	.....	129 000	3.33	585	(20)
			....	1915	.....	108 000	3.48	512	(20)
			....	1916	.....	77 600	3.09	327	(20)
			....	1917	.....	115 000	3.66	573	(20)
			....	1918	.....	94 300	3.66	469	(20)
			....	1919	.....	111 000	3.13	473	(20)
			....	1920	.....	87 800	3.15	376	(20)
			....	1921	.....	93 500	3.00	382	(17)
			....	1922	.....	115 000	3.52	551	(17)
			....	1923	.....	109 000	3.19	473	(17)
			....	1924	.....	111 000	2.91	440	(17)
			....	1925	.....	89 500	3.16	384	(17)
			....	24 years, total mean	.....	2 540 000	3.05	10 500	
167	Sutlej..........	Head of Sirhind Canal, India............	....	Aug-Dec., 1893	47	....	1.15	....	(28)
			....	Jan-Dec., 1894	128	....	2.86	....	(28)
			....	Jan-Oct., 1895	85	....	2.00	....	(28)
			....	Apr-Oct., 1896	49	....	2.82	....	(28)
			....	June Aug., 1897	22	....	5.45	....	(28)
				(*k*) STREAMS IN CHINA					
168	Yangtse........	Hankow............	....	3/11-11/8, 1923	14	530 000	0.53	380	(68a)
169	Yangtse........	Wuhu...............	....	1/28-10/30, 1924	17	520 000	0.27	190	(68b)
			....	Jan.	.....	29 500	0.11	4.42	(67)
			....	Feb.	.....	26 300	0.08	2.87	
			....	March	.....	34 100	0.08	3.72	
			....	April	.....	47 800	0.16	10 4	
			....	May	.....	65 000	0 18	15.9	
			....	June	.....	78 000	0.34	36.0	

* See Appendix.

TABLE 6—(*Continued*)

Item No.	Stream	Locality	Drainage area, in square miles	Period	Number of observations in period	Quantity of water, in thousands of acre-feet	Suspended Silt: Per thousand	Suspended Silt: Millions of tons (2000 lb.) during period	Reference number *
				(*k*) Streams in China (*Continued*)					
169	Yangtse......	Wuhu..............	....	July	.....	101 000	0.74	101	
			....	August	.....	106 000	0.38	55.0	
			....	Sept.	.....	94 000	0.47	60.0	
			....	Oct.	.....	84 000	0.46	52.7	
			....	Nov.	.....	63 600	0.28	24.3	
			....	Dec.	.....	43 500	0.13	7.65	
				Mean year, Item No. 169	.....	773 000	0.36	374	
170	Huang Ho (Yellow R.)......	Chiang-Kou, China...	....	5/1–9/30, 1919	.....	23 300	39.5	1 250	(12)
171	Si Kiang (West).	Wuchow............	....	7/6–10/13, 1915	9	....	0.44	....	(1); (20)
				(*l*) Streams in Australia					
172	Murrumbidgee..	Burrinjuck..........	5 000	7/1–6/30, 1910–11	.....	810	0.15	0.162	(58)
			....	1911–12	.....	440	0.24	0.142	(58)
			....	1912–13	.....	840	0.40	0.450	(58)
			....	1913–14	.....	725	0.94	0.930	(58)
			....	1914–15	.....	386	0.37	0.195	(58)
			....	1915–16	.....	1 020	0.40	0.560	(58)
			....	Total mean....	.....	4 220	0.42	2.44	

* See Appendix.

Follett used a weight of 53 lb per cu ft for silt in the reservoir. If this specific weight is correct, 187 000 acre-ft of suspended silt passed San Marcial, which is more than was deposited in the reservoir. It is believed that only a negligible quantity of silt passed through the reservoir. On the basis of 65 lb per cu ft the volume of silt passing San Marcial amounted to 153 000 acre-ft of suspended silt, leaving 25 000 acre-ft for that brought in as bed load. This is 16% of the suspended load. It is thus seen that, in this case, the estimated volume of the bed load hinges on what specific weight is adopted for the reservoir deposits.

The writer investigated the silt carried and deposited by Coeur d'Alene River and from the resulting data made a rough estimate of the bed load. The investigation involved 9 miles of the river above Rose Lake, Idaho, in which twenty-six cross-sections were established and permanently marked. Surveys for silt content were made in May and November, 1921, and in March and July, 1922. Daily water samples were taken above and below this reach. Of the suspended load entering the reach, 75% was fine silt from ore-reduction works in the Coeur d'Alene Mining District and 25% was natural débris. The following data were obtained:

Period (May 13, 1921, to June 30, 1922), in days....	414
Suspended silt entering reach, in tons.............	444 600
Suspended silt leaving reach, in tons..............	363 100
Suspended silt deposited, in tons..................	81 500
Total deposits determined from cross-sections, in cubic yards	190 000

It is believed that 65 lb per cu ft will fairly represent the average specific weight of these deposits. Using this value, the bed load, in cubic yards, is found to be:

Total deposits, from soundings...........	190 000
Suspended load	93 000
Bed load	97 000

From this, it appears that the bed load and suspended load were each equal to about one-half the total. Although this work was done with care considerable uncertainty still remains. The volume assigned to bed load hinges on the specific weight assigned to the deposits. If instead of 65 lb per cu ft, a value of 80 lb is used, the bed load becomes 115 000 cu yd, or 60% of the deposits measured in place. If 50 lb is used, the bed load becomes 36% of the measured deposits.

Again, if the sampling gave results 10% too high at the upper end and 10% too low at the lower end, the entire volume deposited would have to be accounted for by bed load. If these errors were reversed the entire deposits would have come from the suspended load.

It would be better to express bed load as a percentage of the total load when possible. The foregoing data on bed load may be summarized in percentage of total débris transported, as follows:

	Suspended load	Bed load
Mississippi River, at mouth..................	90	10
Rio Grande, at San Marcial, N. Mex..........	86	14
Colorado River, at Yuma, Ariz. (Fortier and Blaney)	80	20
Coeur d'Alene River, at Rose Lake, Idaho....	49	51

The foregoing data are to be considered only as the roughest approximations; some are pure estimates, but they constitute the only data known to the writer. The difficulties of differentiating between bed load and suspended load are well illustrated in the case of the Coeur d'Alene River. Strictly, there is no line of demarcation—one merges indistinguishably into the other. As far as known no successful attempt has been made to measure the material transported in a river as bed load.

Laws of Silt Transportation.—Ordinary river flow is turbulent. The moving water prism contains many eddies and cross-currents. These serve to maintain the finer material in suspension and roll the coarser along the bed. The same material may alternate as bed load and suspended load. Bellasis (15*a*) defines suspended load as "silt", bed load as "drift"; and the weight of material per unit volume of water as the "charge".

There is an upper limit to the quantity of material a stream of given depth and velocity can transport. Such a charged stream will deposit silt at every reduction of velocity or decrease of turbulence and pick up more, if available, at every increase. A fully charged stream can not scour material from its channel, but its power of moving the drift or bed load is not impaired (15*b*).

About 1900, R. G. Kennedy, Executive Engineer, Public Works Department, Punjab, India, advanced the first formulas for transportation of silt by water as a result of his study of the Bari Doab Canal System. Kennedy presented the theory that there is a critical velocity for each depth that will neither scour nor permit deposits. His general formula is:

$$V_0 = C\, d^n \qquad (1)$$

in which, V_0 is the critical velocity that will neither drop nor pick up sediment; d is the depth of water; and C is a coefficient depending on the kind of silt.

The values of the constants in Equation (1) have been established experimentally (see Table 7).

TABLE 7.—CONSTANTS FOR SUBSTITUTION IN EQUATION (1)

C	n	Authority	Applicable to:
0.84	0.64	Kennedy	Upper Bari Doab Canal, Punjab, India.
0.91	0.57	Kennedy	Shwebo Canal, Burma.
0.67	0.55	Kennedy	Godaveri, Western Delta, Madras, India.
0.93	0.52	Kennedy	Kistna, Western Delta, Madras, India.
0.95	0.57	Lindley	Lower Chenal Canal, Punjab, India.
0.39	0.73	Ghaleb	Egypt.

Mr. Gerald Lacey presented the first general treatment of this complex subject in 1929 (57). Lacey[4] advances the theory that a stream flowing on its own alluvial plain possesses the following quite remarkable characteristics:

1.—The cross-section on straight reaches tends to become semi-elliptical.

2.—The parameter of the ellipse (ratio of the major to one-half the minor axis or surface width to maximum depth) depends solely on the character of the silt as regards fineness.

3.—For a given discharge, the wet perimeter is constant and independent of the character of silt.

4.—The silt factor bears a definite relation to the roughness coefficient in the Kutter and Manning formulas for discharge.

All attempts to apply the Kennedy formulas to the Imperial Valley conditions have proved unsuccessful (42*e*), and it appears that the Lacey formulas will prove equally disappointing.

It is scarcely possible that the complex laws of sediment transportation and deposit can be expressed for all kinds of river débris from dust to boulders as simply as Lacey has done it.

The writer is quite familiar with the Platte River in Nebraska. It flows on a deep bed of pure sand. The cross-section is anything but elliptical, and the channel conforms to no law except that of the mythical "Powder River" of the 91st Division of the American Expeditionary Force, whose slogan was: "It's a mile wide and an inch deep; we can swim it!"

[4] The original paper on file in Engineering Societies Library presents the Lacey formulas with considerable data and discussion.

Effect of Clarifying a Silt-Laden Stream.—This promises to become a most important phase of the silt problem. The Boulder Reservoir will discharge clear water for the first time in untold ages into an alluvial river channel. Moreover, the discharge will be much more uniform than now obtains. What effect this will have on the regimen of the river, and how it will affect diversions into canals will be watched with great interest. Changes in the river channel of a silt-laden stream, after its regimen as regards flow and silt content has been radically altered, have been exemplified in the Rio Grande below Elephant Butte Dam. In the 125-mile stretch below this storage dam there are four dams diverting water into six irrigation canals (6*b*). The changes in river regimen that have occurred since the storage reservoir began operating in 1915, are listed in Table 8.

TABLE 8.—Changes in Regimen of Rio Grande

Item No. (1)	Point of measurement (2)	Period of observation (3)	Years (4)	Before construction of Elephant Butte Dam (5)	After construction of Elephant Butte Dam (6)
	(*a*) Mean Annual Run-off, in Acre-Feet				
1	San Marcial, N. Mex.	1897–1914	18	1 150 000	....
2	San Marcial, N. Mex.	1915–1931	17	....	1 510 000
3	E. Paso, Tex.	1897–1914	18	812 000	....
4	El Paso, Tex.	1915–1950	16	....	638 000
	(*b*) Maximum Discharge, in Cubic Feet per Second				
5	San Marcial, N. Mex.	Oct. 11, 1904	....	33,000	....
6	San Marcial, N. Mex.	Sept. 29, 1929	....	....	33 000
7	E. Paso, Tex.	June 12, 1905	....	23 700	....
8	El Paso, Tex.	Sept. 3, 1925	....	....	9 160
	(*c*) Mean Annual Silt Content, in Tons				
9	San Marcial, N. Mex.	1897–1914	18	22 800 000	....
10	San Marcial, N. Mex.	1915–1931	17	....	21 700 000
11	El Paso, Tex.	1906–1909	....	17 500 000	....
12	El Paso, Tex.	1916–1925	....	....	550 000

The general effect on the river channel has been to flatten the slope as indicated by cross-sections of the river taken since the storage began (1915 to 1925).

For the first 100 miles, the river has disposed of the débris from tributaries and, in addition, has taken about 480 acre-ft (2 100 000 cu yd) from its bed and banks and deposited them in the lower reaches. This degradation must continue until the river channel consists of a series of slopes between the diversion dams or other natural obstructions, just adequate to pass the mean-water and silt discharges under the new regimen that obtains under storage control.

On the Colorado River after the Boulder Reservoir is in operation a flattening of the river slope may be expected because large floods will be no more. Material will be picked up from the river channel by the clear water from the reservoir and deposited in the lower reaches. In time, the finer silts in the upper parts of the present channel may be carried entirely away,

leaving only the coarser sands that may then move mostly as a bottom load in a fairly clear stream, except when it is muddied by floods on the tributaries.

Research on Silt Transportation.—The laws of silt transportation are known only imperfectly. They are now being made the subject of intensive research in both America and Europe. The first experimental study of the laws of sediment transportation with particular reference to bed load was undertaken by Gilbert (77) about 1914. His laboratory consisted of flumes with glass sides in which varying quantities and sizes of sands were introduced into streams of varying velocities. He observed transportation by saltation, the phenomena of sand ripples or dunes, and the conditions under which they migrated up stream or down stream. He developed equations for the tractive force to move sand mixtures. The work of Gilbert has not been followed up until recently.

Interest in this problem has again been stimulated and model experiments, together with mathematical studies of stream dynamics, are being made. MacDougall (71) is beginning where Gilbert left off, experimenting to determine the laws of bed-load transportation. Vogel (76), Matthes (75), and a staff of experimenters at the United States Waterways Laboratory at Vicksburg, Miss., are working extensively with models of particular reaches of the Mississippi and other rivers, studying the silt-transportation problems as regards shoaling, scouring, building of bars, effect of bends, etc.

The experimental and mathematical work by Rehbock, Prandtl (72), Hans Kramer, Assoc. M. Am. Soc. C. E., Bulle, and others in Germany, a mathematical review by O'Brien (74), and studies from the geological angle by Rubey (73) on the movement of débris as related to the conservation of energy in river systems, are all adding greatly to the sum total of the knowledge concerning the phenomena of sediment transportation and turbulent flow which are inseparably bound together.

These studies are incomplete, and discussion regarding the significance of observed phenomena is still rife.

Control of Silt

Except on certain small reservoirs for municipal or industrial purposes, it is generally impracticable to remove any substantial quantity of silt from reservoirs after it has been deposited. The most practicable remedy lies in preventing permanent deposits. Under certain circumstances this is quite possible. Some examples in which effective measures have been taken to prevent such permanent silt deposits will be cited.

The Aswan Dam is provided with sufficient sluice-gate capacity to pass the entire flood discharge of the Nile. These sluice-gate are opened at the beginning of the flood period, and the river flows through the reservoir practically as if no dam existed. During such flood periods, although the Nile is heavily charged with silt, no depositions occur in the reservoir. After the peak of the flood has passed, and the river begins to run clear, the sluice-gates are gradually closed and the reservoir is filled for use during the subsequent irrigation season. By this method of operation silting of the Aswan Reservoir has been so far, and probably will be, entirely avoided.

In Algeria, on the Habra and Hamiz Reservoirs, considerable success has been attained by opening sluice-gates of relatively small discharging capacity in the dams at the close of the irrigation season and allowing the stream to cut through the silt deposits, carrying out substantial quantities.

Bhatgurk Reservoir (*5c*) on the Yeluand River, in Bombay, India, completed in 1892, is provided with twenty under-sluices, each 80 sq ft in cross-section. The operations of these scouring sluices have no appreciable effect on silt already deposited, but having sufficient capacity to pass the average flood, the greater part of the silt is carried off while it is yet in suspension.

In the Zuñi Reservoir there were three 14-in. outlet gates in the tower, two of which became useless by silting. The outlet tunnel through the dam is 6 ft. in diameter. In 1931, a 4 by 6-ft sluice-gate was installed in the gate tower at Elevation 950. When first opened there immediately followed a flow of water and silt under a 40-ft head sufficient to fill the tunnel. From July 1, to October 15, 1931, 500 acre-ft (807 500 cu yd) of silt were sluiced out of this reservoir.

Silt Control on the Water-Shed.—Streams draining areas in arid climates, where fine sedimentary materials occur, are always silt laden. Many such streams are ephemeral; that is, they flow during storm periods only. Their valleys are of alluvial deposits that may be built up during long periods of years and then appear to suffer a fairly rapid degrading, the causes of which are complex and not fully understood. Among such streams are the Zuñi, Pecos, Puerco, Chaco, and Gila Rivers.

It is quite well established that a good growth of grasses is very effective in holding the soil. Where the precipitation is sufficient to maintain forests, extensive soil removal is prevented during ordinary amounts of precipitation. During heavy rains, causing extreme floods, neither grasses nor forests can prevent extensive erosion.

On the Zuñi River water-shed, silt control was begun in 1923 (*40i*). Nutria Creek was found to be supplying most of the silt, and systematic protective measures on this tributary were undertaken. Brush and rock checks were built across the main and tributary arroyos at critical points sufficiently close together to form a new and flatter gradient for the streams. At sharp bends and elsewhere at critical points rock and brush mattresses were constructed to prevent excessive cutting and consequent bank caving.

This work has been continued. Each year additional protective works have been built and the old structures maintained. Fig. 2 shows the silt deposits in Zuñi Reservoir. Note the diminution in the rate of silting after protective measures were undertaken in 1923.

The Rio Puerco is one of the main silt-bearing streams tributary to the Rio Grande above Elephant Butte Reservoir. This water-shed has been the subject of special study on behalf of the Middle Rio Grande Conservancy District (10). In former years, this river had a discontinuous channel, the valley floor having a goodly portion of broad, grassy or marshy areas over which moderate flood waters passed in thin sheets. In other places there was

a definite channel, about 15 ft deep and 100 ft, or more, in width. About 1885, the river began a progressive degrading of its channel from its mouth up stream. An arroyo with many tributaries now exists well toward the head-waters. Surveys show that this main arroyo is 150 miles long and averages 28 ft in depth and 285 ft in width. The original channel had only about 12% of the volume of the present channel. These arroyos are being widened by caving of banks and transportation of the silt into the Rio Grande and on into Elephant Butte Reservoir. It has been estimated that during the 42 years prior to 1927 a total of 395 000 acre-ft of silt has been eroded from the valley floor of Rio Puerco and its tributary channels. This is an average of 9 400 acre-ft per yr and is nearly one-half the average deposits in Elephant Butte Reservoir. Protective measures have been planned, but to date have not been undertaken.

Excluding Silt from Canals.—The problem of preventing débris from entering canals diverting from silt-bearing streams has occupied the serious attention of engineers the world over.

On streams that run clear at ordinary stages, but carry a coarse bed load, the best solution has been found to construct the canal gates parallel to the shore of the stream and provide them with flash-board or over-flow gates so that the water taken into the canal is skimmed off the surface. If the water is raised by a dam, a sluice-gate is provided so that débris deposited in front of the gates can be sluiced away and passed on down the river whenever a surplus of water is available for sluicing.

On streams that carry considerable suspended silt in addition to bed load, settling basins have been used—located immediately below or in conjunction with the head-works, with provisions for periodically sluicing the deposits back into the river. One of the best examples of this method of silt control is afforded by the head-works of the Fort Laramie Canal, of the U. S. Bureau of Reclamation, at the Whalen Diversion Dam on the North Platte River, in Wyoming (6*d*).

All-American Canal.—On streams heavily laden with fine suspended silts, such as the Colorado River, desilting is a serious problem. On account of the slowness with which this silt settles even in still water, enormous settling basins are required. Studies are now (1934) under way for the head-works of the All-American Canal from the Lower Colorado River. The intake structure is planned to be located about 19 miles above Yuma, Ariz. Enormous desilting basins have been included in the preliminary designs.

S. L. Rothery, M. Am. Soc. C. E., has advanced (27) the theory that, in this stream, exclusion of the bed load from the canal up stream is of greater importance than exclusion of the suspended load.

The Laguna Dam, on the Colorado River, was completed in 1909. The head-works follow closely the design for silt-laden streams developed in India and Egypt. Three similar dams on the Nile River had been constructed during the fifteen years preceding the design of the Laguna Dam. The dam is 19 ft high, 4 800 ft long, and raises the water 10 ft at low water. It backs water about 10 miles up stream, creating a settling basin. This basin silted

up soon after the dam was put in operation. At the ends of the dam a long sluice-way is provided which takes water from this basin. A long skimming weir with crest flash-boards in the land side of this sluice-way forms the head-gates proper, of the canal. The end of the sluice-way is provided with three large gates of sufficient capacity to pass 20 000 cu ft per sec. Fig. 4 is a general plan of the structure.

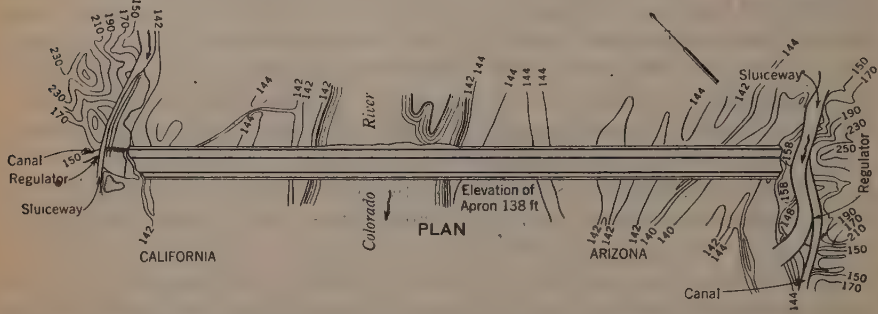

FIG. 4.—PLAN OF LAGUNA DAM.

Experiments on the efficiency of the desilting works at Laguna Dam during August and October, 1918, showed that the water in the main canal at the head contained 57% less silt than that of the river above the dam (42*f*).

Canals in India.—Much study and experimentation have been devoted to methods of preventing silts being taken into the large canal systems from the heavily silt-laden streams of India. In general, diversion is by means of a low dam or barrage. Several canals may head from one diversion dam and generally from both sides of the river.

A stream in alluvium consists of bends, straight reaches, and inflection points between bends. The latter are also called "crossings". At a bend the top flows toward the concave bank and the bottom filaments flow toward the convex bank. The effect of this phenomenon is to cause the bed load to be drawn to the convex side of the bend. A canal, therefore, that heads on the concave side of a bend will receive less bed load than one heading on the other side.

The Indian practice is to construct a dividing wall extending up stream from the barrage and reaching above high water. This wall forms an approach channel to the canal intake. Fig. 5 shows a plan of the Ferozepore Barrage, on Sutlej River. Two dividing walls are provided. Under-sluices are sometimes placed in the dividing wall opposite the canal gates in order to draw the bed load of sand away from the intake.

Passing Silt on to the Land.—Silt that passes into the canal at the headworks must be cleaned every year from the canal system unless the distributaries are designed and constructed to carry the silt through them to the land irrigated, or to discharge it through waste channels. The laws of silt transportation have already been discussed. The practice in India appears to be to pass as much silt as possible on to the lands. Suffice it to

state, thus far, in the United States, the behavior of canals is so erratic and the laws of silt transportation are so imperfectly understood that no one has succeeded in designing a canal system for the Western silt-laden rivers that makes it possible to pass the silt on to the lands. Certain reaches of certain canals have been observed to be practically non-silting and non-scouring for flows near the maximum. For lower flows that must be carried during a part of the season, however, extensive silting inevitably results.

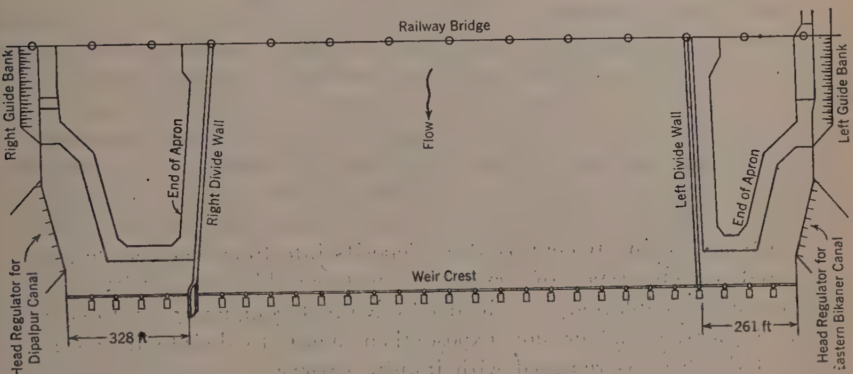

FIG. 5.—FEROZEPORE BARRAGE, SUTLEJ RIVER, INDIA.

The Imperial Irrigation District occupies a part of the delta of the Colorado River. The main canal diverts from the river at the Rockwood Heading, a short distance above the International Boundary. This structure is of concrete, built parallel to the river bank. It consists of seventy-five openings controlled by flash-boards, by which the river surface may be skimmed, thus excluding a part at least of the heavier material. There is no dam except a low brush-and-rock affair built during some years in the low-water period. Silt that enters the canal is removed by dredging. The canal diverts between 2 000 and 6 000 cu ft per sec from the river during the irrigation season. From the head-works of this canal 15 000 000 cu yd of silt were removed by dredging during the three seasons, 1918–1920 (50*b*).

The suspended silt found throughout this Canal System varies from 2 to 8 per 1 000 by weight during the height of the irrigation season. Practically all the suspended silt is so fine that it will pass a 200-mesh sieve, and the greater part passes a 300-mesh sieve (42*g*). A velocity of $\frac{2}{3}$ ft per sec will maintain this silt in suspension.

Experience in the Imperial Valley seems to prove that it would be cheaper to remove the silt from the Canal System by machinery than to pass it to, and care for it on, the cropped lands (42*g*). The quantity of silt removed from the Canal System by machinery is approximately 4 250 000 cu yd per annum (58), of which one-third is taken out by dredges at the head-works. This is a large quantity of material to handle each year, but it is a relatively small part of the amount taken into the system. In 1914, 30 000 000 cu yd of sediment were taken in at the Hanlon Head-Gate. About 4 000 000 cu yd were removed by machinery from the System, leaving 26 000 000 cu yd

to be carried on to the land or passed through wasteways. It is impossible to hold the silt in the Canal System from which it could be dredged cheaply.

The disposal of silt in irrigation systems supplied by the Colorado River and similar silt-laden streams constitutes a serious problem, for which no adequate solution has as yet been found.

Origin of Silt

Exposed rocks disintegrate into soil by varied and complex mechanical and chemical processes. Rain water dissolves certain constituents; it enters crevices, freezes, and spalls off blocks and particles. Chemical agents in the air re-act with those of the rocks, causing them to break down and even to crumble into dust. Wind, rain, and streams transport the soils thus made to lakes and oceans, sorting and depositing them in layers or beds sometimes thousands of feet in thickness. Infiltration of limes or other cementing agents, great pressure from superimposed beds, intense heat, volcanic activity, and subsequent cooling re-convert these deposits into limestones, shales, sandstones, and various types of crystalline metamorphic rocks with infinite gradations and modulations as effected by a great complexity of forces and factors. Upheavals and erosion expose them again to disintegrating influences, and the cycle is repeated with infinite variety.

Sedimentation is that phase of the geological cycle involving water transportation and deposition. Every rill and river performs its allotted part in this process. Lakes and valleys are filled, great deltas extend fan-like into oceans, creating alluvial plains. The densely populous plains of the Nile, Ganges, Mississippi, and Yellow Rivers are the results of such sedimentation processes.

Most streams run fairly clear at low stages, but while so doing they may move quantities of sand and detritus along their beds. At high stages a suspended silt load develops and the stream is said to run "muddy". At every slackening of the velocity both suspended load and bed-carried material are deposited, forming bars, berms, deltas, valleys, and alluvial plains.

Some rivers carry a suspended load at all times. Generally, these rivers drain arid or semi-arid areas. It appears that in areas of abundant precipitation the streams are capable of carrying off all the detritus resulting from rock disintegration as fast as it accumulates. The result is that the streams run clear except at times of flood. On arid areas the débris from disintegration may remain in place for many years until an unusual rainfall (cloudburst) occurs, when great quantities of silts are transported. If the drainage area is large enough and has many tributaries, the "unusual" rainfall is almost continuously occurring successively in some places. This keeps the main stream continually supplied with an abundance of silt, which it must carry at all stages. Of such, are the water-sheds of the Rio Grande, the Colorado, Missouri, Colorado River of Texas, Yellow, and Indus Rivers.

The character of the drainage area and its vegetable covering are all-important factors. If the rocks of the area are sedimentaries, such as sandstone, clays, and shales, disintegration processes produce large quantities

of fine soils. If forests exist this soil is effectively held in place against ordinary rains, but the forest rapidly loses its efficacy to prevent erosion in times of intense rainfall. If the area is too dry for forests, the soil may still be held effectively by substantial growths of grasses and small brush. If too arid for small growth, the soil lies at the mercy of every shower.

Bank cutting is a fruitful source of sediment. The process of moving soil from the mountains to the sea consists of an infinite number of starts and stops. A bar is formed during this flood that may not be moved for many years. An alluvial valley is built up during centuries of sedimentation and then deleted under a new set of cultural or climatic conditions. Alluvial streams build their beds and banks higher than the surrounding plain, a process which, however, can not continue indefinitely. A flood breaks the banks and the river finds a new channel, cutting out the deposits it had itself laid down in earlier years. The Yellow River built up its channel until the water surface was 25 ft above its plain. During the floods of 1851 to 1853 it broke its bank, inundated 50 000 sq miles of cultivated valley, snuffed out a million lives, and found a new mouth 500 miles to the north of its former outlet. This river of mud is often referred to as "China's Sorrow" (12*b*).

The Missouri River is always muddy in its lower reaches. Its Indian name means "Big Muddy". It receives much silt from the clay beds of the Bad Lands of South Dakota and from the enormous areas of shale in Montana and elsewhere in its upper valley, and, in addition, takes continuous toll from its bank through the alluvial plain through which it flows.

The Colorado River becomes a silt-laden stream after it passes into Utah. Much of its drainage area is so arid it is quite void of vegetation. Floods from local rains occur in great diversity on its many ephemeral tributaries that keep the main stream loaded with silt. These characteristics also apply in varying degrees to the Rio Grande, the Colorado River of Texas, the Brazos, Trinity, and many other Southwestern streams.

The Yellow River in Northern China carries nearly twice as much silt per annum as the Mississippi. Its drainage area is largely covered with loess, a yellowish, friable, wind-blown deposit of fine sands and dust-like soil that is carried away with every rainfall, keeping the streams continually surcharged. Its name and that of the Yellow Sea into which it flows is derived from the color this loess gives to the water.

In India, all the great rivers are silt-laden streams. Between the Himalayas and the Indian Peninsula lie the great alluvial plains watered by three great river systems. The Indus, with its principal tributary, the Sutlej, waters the western portion; the eastern portion is drained by the Brahmaputra; while between them lies the Ganges.

The Indus Valley is quite arid and is covered with fine alluvial silts and wind-blown sands. The Ganges and Brahmaputra areas have a seasonal climate. Parts of these areas are seasonally dry; irrigation is profitable. From these areas enormous quantities of silts are supplied to the rivers so that the streams flow on broad alluvial deposits, which although about 2 000 ft thick, are nevertheless geologically recent.

The Nile is perhaps the most famous river of history. The silts deposited on the lands at the time of its annual inundation have made the fertile plains of Lower Egypt. The Nile carries annually 95 000 000 tons of silt, of which 30 000 000 remains on the land and 65 000 000 is carried into the Mediterranean Sea. At ordinary stages, the Upper Nile is fairly clear, but in flood periods it is of a chocolate color from the brown soils of the Abyssinian plateaus brought in by the Sobat, Blue Nile, and Atbara, which streams are the chief sources of the Nile floods.

The Rio Puerco drains 5 700 sq miles of Western New Mexico. The name means "dirty river". It and its neighbouring stream, Rio Salado, are sources of abundant quantities of silt in the Rio Grande and pour these silts into that stream above Elephant Butte Reservoir. Because of this fact Rio Puerco has been the subject of special study from a geological standpoint by Kirk Bryan, Professor of Geology, Harvard University, and from an engineering point of view by Mr. George M. Post (10). Because their report contains data that are essentially applicable to many other streams, they will be presented as quite typical of the silt-bearing ephemeral streams of the arid Southwest.

The silts move only during floods— a quick rush of muddy water down a dry channel immediately following a rain. Rio Puerco has always carried large quantities of silt into the Rio Grande, but this quantity has been increased materially in recent years. The investigation was prompted by the hope and belief that the channel would be rehabilitated and the silt charge greatly reduced.

When first known, the main stream and its tributaries were in a process of alluviation. The valleys were plains over which muddy floods spread out or flowed in discontinuous channels. In places, these channels may have been 10 to 20 ft deep and 100 ft, or more, wide, but they were interspersed by broad, flat, marshy areas over which the water spread in sheets. About 1885, a new arroyo began to form at the mouth and to cut up stream, until the channel is now continuous for 150 miles and averages 28 ft in depth and 285 ft in width.

In order better to understand this process, Figs. 6 and 7 have been reproduced (10), showing how the alluvial valley was formed and how degradation now appears to be progressing in the Valley of Rio Puerco.

The silts come mainly from the disintegration of Cretaceous rocks that cover about one-half the drainage area. The characteristic topography of the silt-producing areas consists of mesas underlaid by sandstone lying between broad valleys eroded in the shales (Fig. 6). The sandstone cliff is undermined by the more rapidly eroding shale and breaks off along joint planes. These pieces weather into sands, while the shale base forms clays. The valleys become filled with fine argillaceous sands that are eroded easily unless held by grass sod or other vegetation. Gullies are formed on the steep hillsides that end in deltaic fans formed by silts deposited at the valley floor. These deposits form cracks when dried, creating vertical joint planes in the mass. When such deposits form the banks of a stream, they

Topographic Divisions	Mesa	Cliff Edge	Hill Slope	Alluvial Fan	Valley Floor	Channel Trench (Arroyo)
Vegetation	Pine, Juniper Piñon		Juniper, Grass	Grass and Sage	Grass, Weeds, Greasewood	Usually Barren Grass and Weeds
Rate of Weathering	Slow	Rapid	Rapid	No Importance	No Importance	
Rate of Run-off	Feeble	Rapid	Rapid	Small Except from Heavy Rains	Rapid	
Rate of Silt Production	Moderate	Moderate	Moderate	Small Unless Gullied	Small Unless Gullied	Excessive, Due to Lateral Erosion

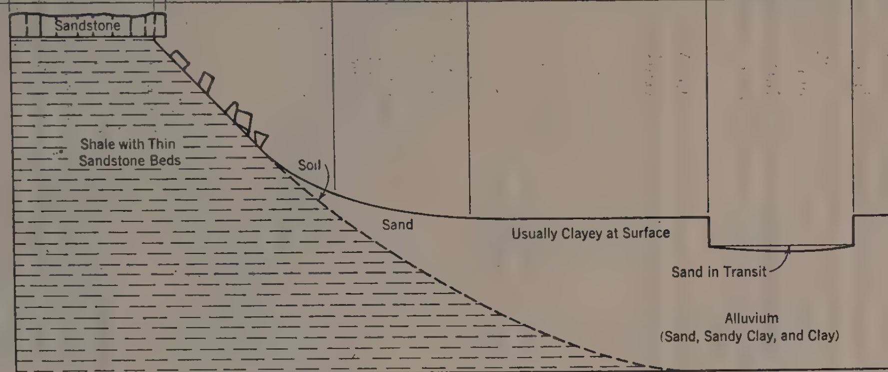

FIG. 6.—DEMONSTRATING HOW ALLUVIAL VALLEYS ARE FORMED.

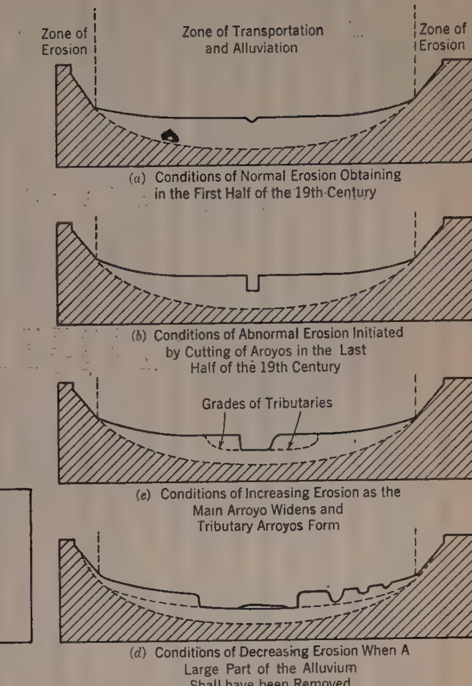

FIG. 7.—DEMONSTRATING HOW AN ALLUVIAL VALLEY MAY BE DEGRADED.

are easily undermined; the banks cave off in blocks, crumble, and become the suspended load of the stream.

Rio Puerco began degrading its valley floor about fifty years ago. Fig. 7 shows the successive events as they have been pieced together by historical inquiry and geological evidence. The silt derived from the valley walls was carried over the surface and on down stream through discontinuous channels, intermittently by the floods. This condition is shown in Fig. 7(*a*). With the cutting of the arroyos, the present period of abnormal erosion or degrading process began (Fig. 7(*b*)). The rate of silt production passes through a cycle. When the main arroyo is narrow and increasing in length up stream, the rate of silt production is increasing. When it becomes a continuous channel the widening process begins (Fig. 7(*c*)). At this stage, the rate of silt production has reached a maximum. When the channel becomes so wide that the floods only touch its sides lightly or intermittently, bank caving and consequent silt production begin to diminish.

A time will come when the entire slice of alluvium above the new grades of the main and tributary arroyos shall have been removed (Fig. 7(*d*)). The valley then reverts to the condition shown in Fig. 7(*a*), but at a lower level. A process of alluviation then sets in that tends to build up the valley floor to some higher level. After this has been accomplished, the cycle of degrading will be repeated.

Abundant evidences were found in the Valley of the Rio Puerco of earlier cycles of alluviation and subsequent degrading. Fig 8 is a map of the present

FIG. 8.—ANCIENT CHANNELS SHOW PREHISTORIC PERIODS OF ALLUVIATION AND DEGRADATION

channel superimposed over an ancient one. The present arroyo has nearly vertical banks, 25 ft in height and 180 to 500 ft apart. The outcrops of the buried ancient channel are readily distinguishable because its filling is lighter in color than that of the present valley alluvium. The bed of the ancient channel was about the same level as the present channel and varied from 75 to 220 ft in width.

Traces of other channels were found, but not mapped. These buried channels represent a period of erosion and degrading similar to that going on at present. Each was followed by a period of alluviation, that completely obliterated traces of old arroyos and in general made a plain of the valley floor over which the flood waters could spread and continue the upbuilding of the valley.

Another famous example of a cycle of alluviation and degrading has been found in the Rio Chaco (10*a*), (78). At Pueblo Bonito, an arroyo from 150 to 450 ft wide and 25 to 30 ft deep exists at present similar in all respects to the one on Rio Puerco. This arroyo has been cut since 1860. Pottery and charcoal, to depths of 20 ft, or more, are found in the valley alluvium showing that this alluviation occurred during its occupancy by pre-historic peoples. After Pueblo Bonito and other large pueblos were built and occupied, an arroyo was cut through the valley floor and later filled by alluviation to its original level. Where this buried channel outcrops in the walls of the present arroyo, pottery of very recent occupancy is found, while at similar depths in the main valley fill only pottery of much earlier periods has been discovered.

This ancient arroyo does not coincide with the present one, but is crossed, touched, and re-crossed by it. The ancient channel may be traced for about five miles. It tells the same story of periods of degrading followed by periods of alluviation.

These ancient peoples probably subsisted largely by "flood-water farming". When the ancient arroyo was cut in the valley about the end of the Great Pueblo period, it rendered such farming methods impossible. This has been assigned by Professor Bryan as a possible reason for the abandonment of these Indian villages (79*a*). A subsequent period of alluviation filled this arroyo and probably permitted re-occupancy. The present period constitutes another cycle of degradation.

The Zuñi River has been the subject of a study similar to that of the Rio Puerco (10*c*). It is an ephemeral stream, carrying only flood waters and large quantities of silt. The drainage basin consists mainly of plateaus and mesas underlaid by Cretaceous sandstone and shale. Like the Rio Puerco its silt is derived mainly from the erosion of the banks of arroyos cut in the valley fill. This stream is also undergoing a process of degrading its valley floor.

Ancient channels have been found in the Valley of Zuñi River, also, that attest to earlier periods of alluviation and degradation. Fig. 9 is a sketch of such an ancient channel exposed in the high banks of Nutria Creek.

Causes of Valley Degradation.—No complete explanation for these cycles of alluviation and degrading has as yet been found. The potent factors are deep seated and are quite certainly beyond Man's power to alleviate to any great extent, except in certain favored situations. The cause is probably to be found in climatic influences, and the physics of sedimentation as applied to the moulding of stream valleys. During a period of alluviation the slope of the valley is flattened. In time, it becomes too flat for the stream to

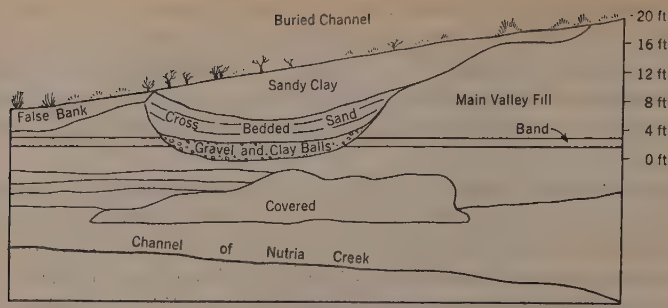

FIG. 9.—ANCIENT CHANNEL EXPOSED IN PRESENT ARROYO, ZUNI RIVER.

carry its normal flood waters. A period of channel cutting ensues. This process of degradation is carried beyond the balance point, and a period of alluviation sets in. Thus, valley building appears to be an endless succession of periods of alluviation and degradation.

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